Pat Hanrahan Stanford University

EuroVis 2009 Keynote

Systems of Thought:

Representations (Symbols) and Rules to Manipulate Them

Language

Systems of Thought

Language

Logic (Boole's *The Laws of Thought*)

Language

Logic (Boole's *The Laws of Thought*) Mathematics

Systems of Thought

Language Logic (Boole's *The Laws of Thought*) Mathematics Probabilistic reasoning and statistics

Language

Logic

Mathematics

Probabilistic reasoning and statistics

Computation

Computational Thinking

Jeanette Wing et al.

Collection of ideas and techniques

- Algorithmic thinking
- Programming
- Systems building

— ...

Language

Logic

Mathematics

Probabilistic reasoning and statistics

Computation

Visualization and visual thinking

Systems of Thought

Language

Logic

Mathematics

Probabilistic reasoning and statistics

Computation

Visualization and visual thinking

Which one is best? (Hint: None) What are the advantages of each?



































Problem Is	omor	ph		
	4	3	8	
	9	5	1	
	2	7	6	
Magic Square: A	All rows	, colum	ns, diag	gonals sum to 15















On Being in the Right Space





On Being the Right Size

"The most obvious differences between different animals are differences of size, but for some reason zoologists have paid singularly little attention to them. In a large textbook of zoology before me I find no indication that the eagle is larger than the sparrow, or the hippopotamus bigger than the hare, though some grudging admissions are made in the case of the mouse and the whale. But yet it is easy to show that the hare could not be as large as a hippopotamus, or a whale as small as a herring. For every type of animal there is a most convenient size, and a large change in size inevitably carries with it a change of form"

J. B. S. Haldane

On Being in the Right Space

"The most obvious differences between different visualizations are differences of space, but for some reason visualization scientists have paid singularly little attention to them. In a large textbook of visualization before me I find no indication that the log-log space is different than the loglinear space, or that the Mercator projection is different than the azimuthal equidistant projection, though some grudging admissions are made in the case of the parallel and perspective projections. But yet it is easy to show that distances are difficult to estimate under perspective, or that data obeying a power law is easy to see in a log-log plot. For every type of visualization there is a most convenient space, and a change into the right space inevitably makes relationships clearer."

P. Hanrahan

The Value of Visualization

The Value of Visualization

The representation effect: Human performance varies enormously (10-100:1) with different representations

The right representation

- Faster solution
- **Fewer errors**
- Better comprehension and memory

—

But, the R. R. depends on the problem/question/task





Key Questions

- 1. What is the problem you are trying to solve?
- 2. How do you think about the problem? What are the semantic objects and their relationships?
- 3. What visual representations are already used? How does the visualization represent those objects and support inference?

How Visual Representations Support Reasoning? "Why is a Picture (Sometimes) Worth 10,000 Words"

Larkin and Simon, Cognitive Science, 1987

Why?

Reduce memory load

- Working memory is limited
- Store information in the diagram

Reduce search time

- Pre-attentive (constant-time) search process
- Spatially-indexed patterns store the "facts"

Allow perceptual inference

Map inference to pattern finding



Number Representations

Counting – Tallying

Adding – Roman numerals

XXIII + XII = XXXIIIII = XXXV

Multiplication – Arabic number systems





Distributed Cognition	
External (E) vs. Internal (I) process	

		Roman	Arabic			
	Separate power & base	I	Е			
	Get base value	E	I			
	Multiply base values	I.	I			
	Get power values	I.	E			
5.	Add power values	I	E			
6.	Combine base & power	I	E			
	Add results	I.	E			
Arabic more efficient than Roman						

Contemplate

Why do computers use binary representations of numbers?

Contemplate

Why do computers use binary representations of numbers?

Who don't people use binary representations of numbers? For example, we use base 10.

What about the abacus?

Abstraction in Computer Science

Abstract data type Choose the interface Different possible representations of the data Running times of key operations depend on representation Choose the appropriate implementation for the problem

Notation as a Tool of Thought

K. Iverson's 1979 ACM Turing Award Address

Notation as a Tool for Thought

"The thesis of the present paper is that the advantages of executability and universality found in programming languages can be effectively combined, in a single coherent language, with the advantages offered by mathematical notation"

K. Iverson

Arithmetic and Algebra in APL (k)





Program Transformations as Proofs

+/ (1 + 2 * til k) // o +/ (1 + (til k) + (til k)) // d +/ (1 + (til k) + (reverse til k)) // a // 0 +/ (1 + k # (k-1)) +/ k#k // k k*k // k // 3

K. Iverson, Arithmetic, 1991

// odd numbers
// def of multiplication
// addition associative
// 0 1 2 + 2 1 0 = 2 2 2

// k = k-1+1 // k*k = +/ k#k // 3*4 = +/ 4 4 4

The Incredible Convenience of Mathematica Image Processing

Theodore Gray

Key Questions

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- 4. How can the manipulation of the representation be embodied in the interaction?



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Python Solution A = 0 for i in range(N): x = Uniform() y = Uniform() if x*x + y*y < 1: A += 1 p = A/N return 4*p</pre>











$\begin{aligned} & f(x,y) = x^2 + y^2 < 1 \\ & Z = f(X,Y) \\ & E[Z] = \frac{1}{N} \sum z_i \\ & \text{Central limit theorem for Bernoulli process} \\ & \Pr(E[Z] > z) = N(\frac{E(Z) - z}{\sqrt{Np(1 - p)}}) \end{aligned}$

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- 4. How can the manipulation of the representation be embodied in the interaction?
- 5. How can visualization be coupled with other systems of thought?

Final Thoughts / Bigger Questions

If you can see it, it's a visualization! Simple is complex; abstracting the essence is hard

Language and visualization? What if the problem is not well-defined? What about aesthetics and style?



