Factoring Light Fields

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Radiance Field

$L(r) \quad L(x, \quad )$

Ancients, Ray pyramids, from-eye or from-lights, plenums...
Lambert, Photometria, 1759
Faraday, Fields (lines of force), 1846
Schuster, Radiation transport in planar atmospheres, 1906
Gershun, The light field (irradiance vector field), 1936
Nicodemus, Radiance, 1963
Moon, The photic field, 1981
**Gibson’s Light Field**

From J. J. Gibson, *The senses considered as a perceptual system*, 1966

**Adelson and Berger’s Plenoptic Fn**

Prop. 1. The primary task of early vision is to deliver a small set of useful measurements about each observable location in the plenoptic function.

Prop. 2. The elemental operations of early vision involve the measurement of local changes along various directions of the plenoptic function.

Light as a First-Class Object

Common explicit representations
- Geometry: transformations, polygons, patches
- Colors
- Images

Light represented internally
- Ray tracing (radiance): trace(ray)
- Radiosity (radiant exitance): B(u,v)
- Environment maps (radiance)

Light should also be explicit, or a first-class object
- Lines and visibility
- Energy or Radiometry/photometry

Lines and Light 1

Geometric calculations in line-space
- Camera as a pencil of lines through a point
  - Translated as a set of lines intersecting a line

- The geometry of multiple images [Faugeras & Luong]
  - Fundamental matrix
  - Trilinear tensor
Lines and Light 2

Throughput \( dT \quad d \quad dA \)

Throughput is the natural measure on lines
Throughput conserved in an optical system
Radiance is the ratio of two conserved quantities

\[
L(r) \quad \lim_{T \to 0} \frac{(T)}{T} \quad \frac{d}{dT}
\]

\[
\therefore \text{ Radiance conserved}
\]

Light Field as a 2D Array of Image

Camera Array

\[
L(r) \quad L(u,v,s,t)
\]
Dual Interpretation of Light Field

**Plenoptic Light Field**
- Field radiance

**Surface Light Field**
- Surface radiance

**UV Array of ST Images**

**ST Array of UV Images**

Structure of the Surface Light Field

\[
B(x, \theta) = \frac{1}{n^2} f_r(i, \theta) L(x, R(n(x))) \cos \theta \, d \theta
\]

- **Albedo**
  \( \rho(x) \)

- **BRDF**
  \( f_r(i, \theta) \)

- **Incident Lighting**
  \( L(x, \phi) \)

- **Tangent Frame**
  \( R(n(x)) \quad R(\phi, \phi) \)
Orientation Light Field

\[ B(n, o) \cdot f_i(n, o) \cdot L(n) \cdot \cos \cdot d_i \]

Product of matrices

\[ B[n][o] \cdot R[o][i] \cdot D[n][i][i][i][i] \cdot L[i] \]

Factoring the reflected light field

Inverse Rendering

<table>
<thead>
<tr>
<th>Known Lighting</th>
<th>Known</th>
<th>Unknown</th>
<th>Marschner and Greenberg 97</th>
</tr>
</thead>
<tbody>
<tr>
<td>Known BRDF</td>
<td>x</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| Unknown        | Sato et al. 97  
|                | Dana et al. 99  
|                |Debevec et al. 00  
|                | Marschner et al. 00  |
|                | Sato et al. 99  
|                | Yu et al. 99  |

Textures are a third axis
Spherical Harmonic Analysis

\[ B(n, o) \sum_{i, o} f_r(i, o) L(R(n) i) \cos i d i \]

\[ L(\ ) \quad L_{lm} Y_{lm}(\ ) \]

\[ B_{lm,pq} \quad l \quad l_{pq} L_{lm} \]

Lambertian Reflector

\[ \rho_l = 2\pi \sum (-1)^{l+1} \left( \frac{l!}{(l+2)(l-1)\left(\frac{i}{2}\right)^2} \right) \quad l \text{ even} \]

R. Ramamoorthi and P. Hanrahan, 2001
R. Basri and D. Jacobs, 2001
Inverse BRDF and Lighting

\[
\begin{align*}
\hat{B}_{lmq} & = \sum_l \frac{1}{L_{lm}} B_{lmq} \\
L_{lm} & = \sum_l \frac{1}{\hat{B}_{lmq}} B_{lmq}
\end{align*}
\]

Surface roughness

Angular width of Light Source

Inverse Lambertian

True Lighting  Sum \(l=2\)  Sum \(l=4\)

Mirror

Teflin
Factoring the Light Field

Given: \( B \) find \( L \) and \( \rho \)

\[
B = L \otimes \rho
\]

4D 2D 3D

More knowns (4D) than unknowns (2D/3D)

Light Field can be factored
- Up to global scale factor
- Assumes reciprocity of BRDF
- Can be ill-conditioned
- Analytic formula derived

Summary

Recent results
- SLF is multilinear in albedo, lighting, BRDF and shape (orientation)
- Radiance cannot be recovered from irradiance
- Irradiance accurately approximated by 9 params.
- 4D SLF can be factored into incident radiance and BRDF
- 4D OLF can be efficiently represented as a SHRM

Think lines and light
Forward and Inverse Problems

Computer vision
Inverse problem ill-conditioned
- Output not that sensitive to changes in input
- Cannot robustly recover input parameters
- Inference requires assumptions
  - Regularization or frequency cut-off
  - Priors or models of the environment
- Leads to “constancy”
  - Separating texture and illumination
  - Natural lighting only slightly more complex than directional or point lighting

Forward and Inverse Problems

Computer graphics
Forward problems
- Output not that sensitive to changes in input
- Simplified models of input (lighting)
- Leads to efficient algorithms
  - Hierarchical radiosity
  - Environment and irradiance maps
  - Linear subspace techniques
  - Theoretical basis for image-based rendering
  - Perceptually-based rendering tricks