Factoring Light Fields

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With contributions from

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Radiance Field

$L(r)$ $L(x, \ )$

Ancients, Ray pyramids, from-eye or from-lights, plenums…

Lambert, Photometria, 1759

Faraday, Fields (lines of force), 1846

Schuster, Radiation transport in planar atmospheres, 1906

Gershun, The light field (irradiance vector field), 1936

Nicodemus, Radiance, 1963

Moon, The photic field, 1981

Gibson's Light Field

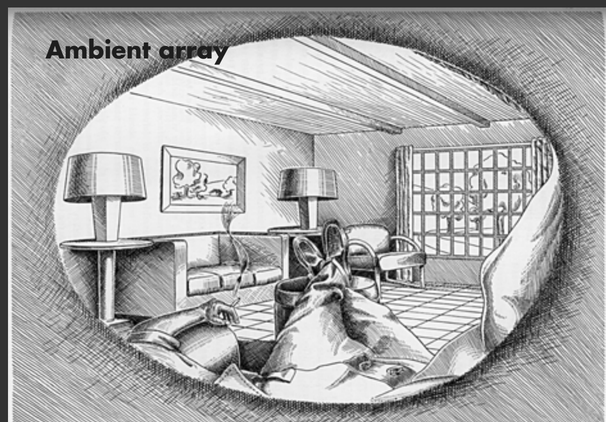


FIGURE 81. The Modern Visual Ego

This drawing, like the one made by Ernst Mach in the 1880's (Fig. 5, ch. 3) represents the visual field of the left eye. The nose, lip, and part of the cheek are visible on the right; the body lies at the bottom of the field; and the environment appears "out there".

From J. J. Gibson, The senses considered as a perceptual system, 1966

Adelson and Berger's Plenoptic Fn

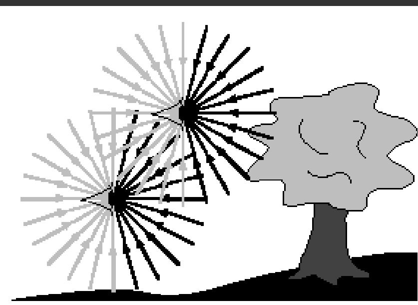


Fig.1.3
The plenoptic function describes the information available to an observer at any point in space and time. Shown here are two schematic eyes-which one should consider to have punctate pupils-gathering pencils of light rays. A real observer cannot see the light rays coming from behind, but the plenoptic function does include these rays.

Prop. 1. The primary task of early vision is to deliver a small set of useful measurements about each observable location in the plenoptic function.

Prop. 2. The elemental operations of early vision involve the measurement of local changes along various directions of the plenoptic function.

From E. H. Adelson and J. R. Bergen, The plenoptic function and the elements of early vision, 1991

Light as a First-Class Object

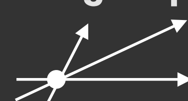
Common explicit representations
- Geometry: transformations, polygons, patches
- Colors
- Images

Light represented internally
- Ray tracing (radiance) : trace(ray)
- Radiosity (radiant exitance) : B(u,v)
- Environment maps (radiance)

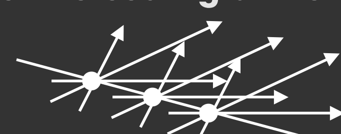
Light should also be explicit, or a first-class object
- Lines and visibility
- Energy or Radiometry/photometry

Lines and Light 1

Geometric calculations in line-space
- Camera as a pencil of lines through a point



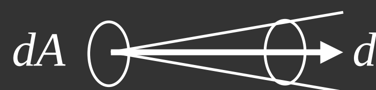
- Translated as a set of lines intersecting a line



- The geometry of multiple images [Faugeras & Luong]
  - Fundamental matrix
  - Trilinear tensor

Lines and Light 2

Throughput $dT \quad d \quad dA$



Throughput is the natural measure on lines

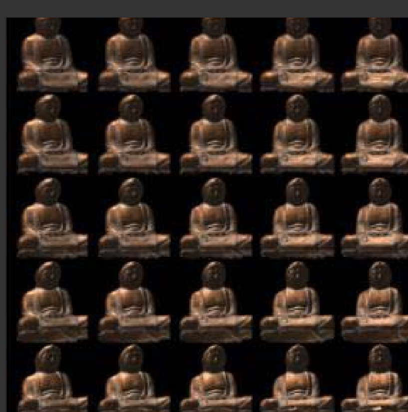
Throughput conserved in an optical system

Radiance is the ratio of two conserved quantities

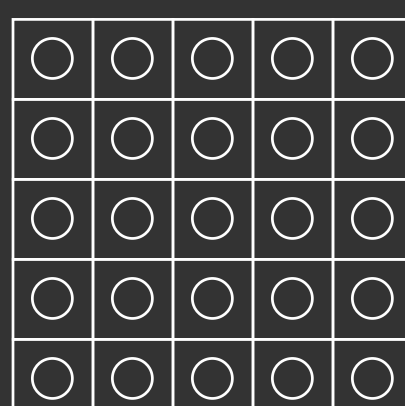
$$L(r) \quad \lim_{T \quad 0} \frac{(\quad T)}{T} \quad \frac{d}{dT}$$

$\therefore$ **Radiance conserved**

Light Field as a 2D Array of Image



Camera Array



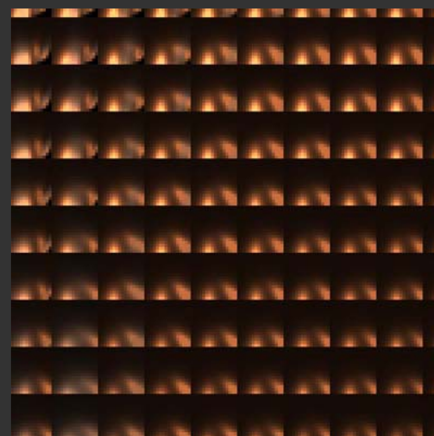
$$L(r) \quad L(u,v,s,t)$$

Dual Interpretation of Light Field

Plenoptic Light Field
Field radiance

UV Array of ST Images

Surface Light Field
Surface radiance



ST Array of UV Images

Structure of the Surface Light Field

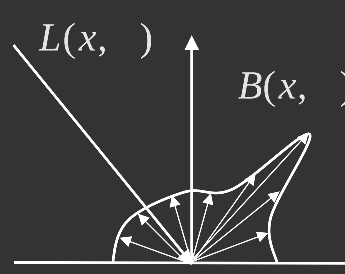
$$B(x, \ _o) \qquad (x) \underset{H^2}{\ } f_r(\ _i, \ _o) L(x, \mathbf{R}(n(x)) \ _i) \cos \ _i \, d \ _i$$

Albedo (x)

BRDF $f_r(\ _i, \ _o)$

Incident Lighting $L(x, \ )$

Tangent Frame $\mathbf{R}(n(x)) \quad \mathbf{R}(\ , \ )$



Orientation Light Field

$$B(n, \;_o) \qquad {}_{H^2}\; f_r(\;_i, \;_o) L(\mathbf{R}(n)\;_i)\cos\;_i d\;_i$$

Product of matrices

$$B[n][o] \qquad R[o][i]D[n][i][i]L[i]$$

$$\mathbf{R}\,\mathbf{D}\,\mathbf{L}$$

Factoring the reflected light field

Inverse Rendering

BRDF \ Lighting	Known	Unknown
Known	X	Marschner and Greenberg 97
Unknown	Sato et al. 97 Dana et al. 99 Debevec et al. 00 Marschner et al. 00	Sato et al. 99 Yu et al. 99

Textures are a third axis

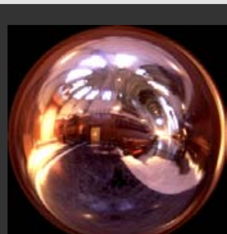
Spherical Harmonic Analysis

$$B(n, \quad _o) \qquad \underset{H^2}{} f_r(\quad _i, \quad _o) L(\mathbf{R}(n) \quad _i) \cos \quad _i d \quad _i$$

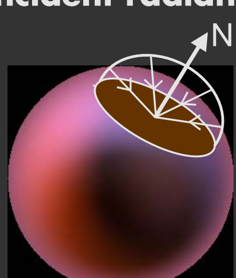
$$L(\quad) \qquad \underset{lm}{} L_{lm} Y_{lm}(\quad)$$

$$B_{lm,pq} \qquad {}_l \quad {}_{lq,pq} L_{lm}$$

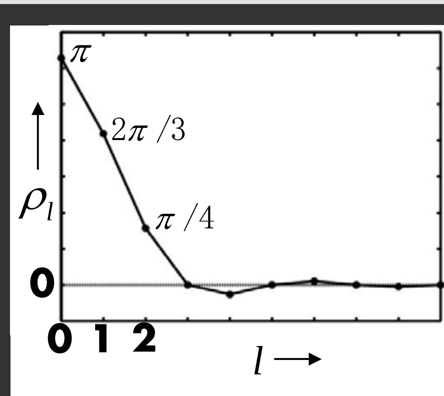
Lambertian Reflector



Incident radiance



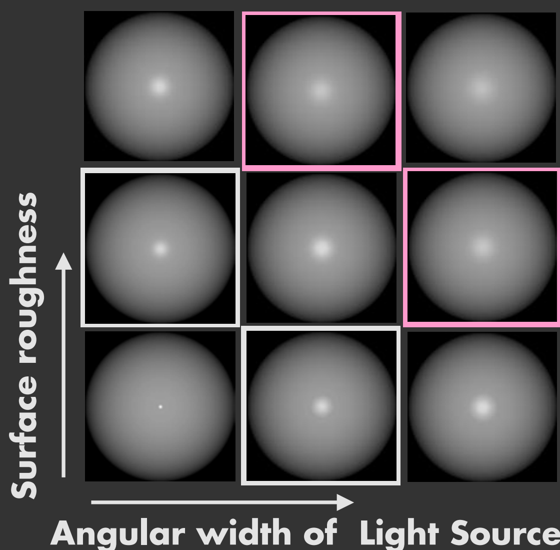
Irradiance



$$\rho_l = 2\pi \frac{(-1)^{\frac{l}{2}-1}}{(l+2)(l-1)} \left[\frac{l!}{2^l \left(\frac{l}{2}!\right)^2} \right] \qquad l \text{ even}$$

R. Ramamoorthi and P. Hanrahan, 2001
R. Basri and D. Jacobs, 2001

Inverse BRDF and Lighting



$$\hat{\ }_{lpq} \quad \frac{1}{\ _{l}} \frac{B_{lmpq}}{L_{lm}}$$

$$L_{lm} \quad \frac{1}{\ _{l}} \frac{B_{lmpq}}{\hat{\ }_{lpq}}$$

Inverse Lambertian

Factoring the Light Field

Given: B find L and ρ

$$B = L \otimes \rho$$

$B \rightarrow$ 4D, $L \rightarrow$ 2D, $\rho \rightarrow$ 3D

More knowns (4D)
than unknowns (2D/3D)

Light Field can be factored

- Up to global scale factor
- Assumes reciprocity of BRDF
- Can be ill-conditioned
- Analytic formula derived

Summary

Recent results

- SLF is multilinear in albedo, lighting, BRDF and shape (orientation)
- Radiance cannot be recovered from irradiance
- Irradiance accurately approximated by 9 params.
- 4D SLF can be factored into incident radiance and BRDF
- 4D OLF can be efficiently represented as a SHRM

Think lines and light

Forward and Inverse Problems

Computer vision

Inverse problem ill-conditioned

- Output not that sensitive to changes in input
- Cannot robustly recover input parameters
- Inference requires assumptions
  - Regularization or frequency cut-off
  - Priors or models of the environment
- Leads to "constancy"
  - Separating texture and illumination
  - Natural lighting only slightly more complex than directional or point lighting

Forward and Inverse Problems

Computer graphics

Forward problems

- Output not that sensitive to changes in input
- Simplified models of input (lighting)
- Leads to efficient algorithms
  - Hierarchical radiosity
  - Environment and irradiance maps
  - Linear subspace techniques
  - Theoretical basis for image-based rendering
  - Perceptually-based rendering tricks