Appendix A

Properties of the Representation Matrices

In this appendix, we derive the two properties of representation matrices listed in equation 2.35. The first property follows from the addition theorem for spherical harmonics (see for instance, Jackson [34] equation 3.62),

$$Y_{l0}(u,v) = \Lambda_l \sum_{m=-l}^{l} Y^*_{lm}(\theta,\phi) Y_{lm}(\theta',\phi'). \tag{A.1}$$

Here, v is a dummy-variable since Y_{l0} has no azimuthal dependence, and u is the angle between (θ, ϕ) and (θ', ϕ'), i.e.

$$\cos u = \cos\theta\cos\theta' + \sin\theta\sin\theta'\cos(\phi - \phi'). \tag{A.2}$$

Now, let $(u, v) = R_\alpha(\theta', \phi')$. Here, $R_\alpha = R_y(\alpha)$. We omit the z rotation since that does not affect Y_{l0} which has no azimuthal dependence. The vector corresponding to coordinates (u, v) is then given by

$$\begin{pmatrix} \sin u \cos v \\ \sin u \sin v \\ \cos u \end{pmatrix} = \begin{pmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{pmatrix} \begin{pmatrix} \sin\theta'\cos\phi' \\ \sin\theta'\sin\phi' \\ \cos\theta' \end{pmatrix} = \begin{pmatrix} \cos\alpha\sin\theta'\cos\phi' + \sin\alpha\cos\theta' \\ \sin\theta'\sin\phi' \\ \cos\alpha\cos\theta' + \sin\alpha\sin\theta'\left(-\cos\phi'\right) \end{pmatrix}. \tag{A.3}$$

Since $(-\cos\phi') = \cos(\pi - \phi')$, we know from equation A.2 that u corresponds to the angle between (α, π) and (θ', ϕ'). In other words, we may set $(\theta, \phi) = (\alpha, \pi)$. To summarize,

$$Y_{l0}\left(R_\alpha(\theta', \phi')\right) = \Lambda_l \sum_{m=-l}^{l} Y_{lm}^*(\alpha, \pi) Y_{lm}(\theta', \phi'). \tag{A.4}$$

To proceed further, we write the rotation formula for spherical harmonics, omitting the z rotation by β, since that has no significance for azimuthally symmetric harmonics.

$$Y_{l0}\left(R_\alpha(\theta', \phi')\right) = \sum_{m=-l}^{l} d_{0m}^l(\alpha) Y_{lm}(\theta', \phi') \tag{A.5}$$

A comparision of equations A.4 and A.5 yields the first property of representation matrices in equation 2.35, i.e.

$$d_{0m}^l(\alpha) = \Lambda_l Y_{lm}^*(\alpha, \pi). \tag{A.6}$$

To obtain the second property in equation 2.35, we use the form of the spherical harmonic expansion when the elevation angle is 0, i.e. we are at the north pole. Specifically, we note that $Y_{lm'}(0', \phi') = \Lambda_l^{-1}\delta_{m'0}$. With this in mind, the derivation is as follows,

$$\begin{aligned} Y_{lm}(\alpha, \beta) &= Y_{lm}\left(R_{\alpha,\beta,\gamma}(0', \phi')\right) \\ &= \sum_{m'=-l}^{l} D_{mm'}^l(\alpha, \beta, \gamma) Y_{lm'}(0', \phi') \\ &= \Lambda_l^{-1} D_{m0}^l(\alpha, \beta, \gamma). \end{aligned} \tag{A.7}$$

This brings us to the second property stated in equation 2.35,

$$D_{m0}^l(\alpha, \beta, \gamma) = \Lambda_l Y_{lm}(\alpha, \beta). \tag{A.8}$$

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