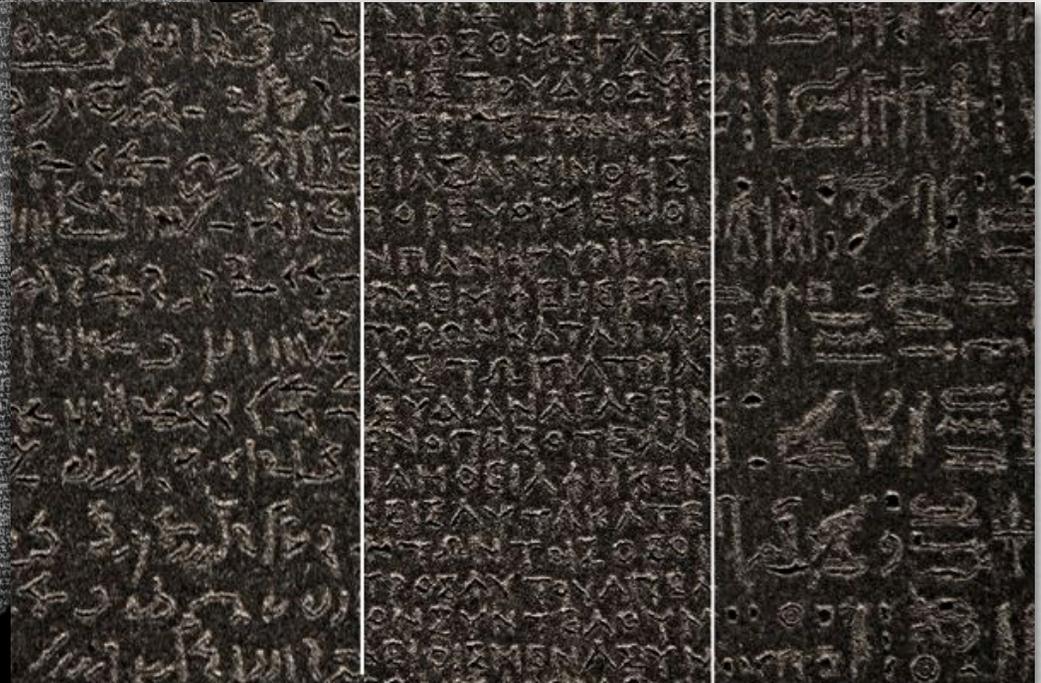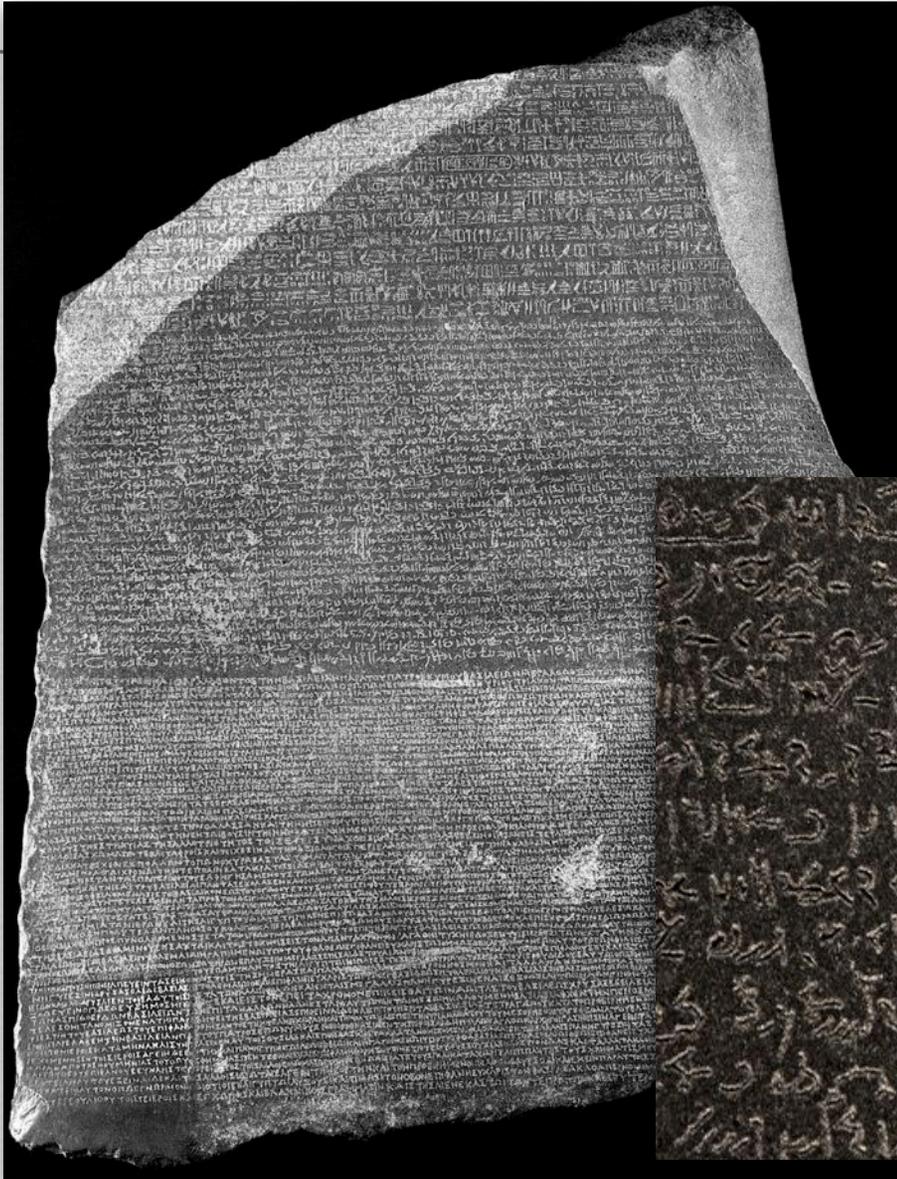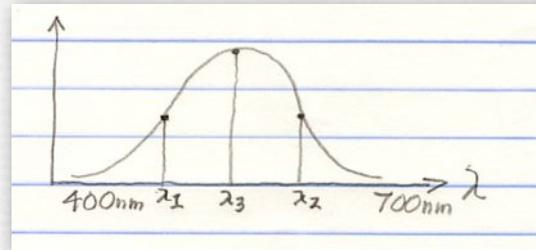# Modern color theory

CS 48N, Winter 2011

Marc Levoy

Computer Science Department
Stanford University

Thomas Young (1773-1829)

Monochromats
# (contents of whiteboard)

1

1. organisms having only one kind of retinal receptor cannot distinguish changes in intensity from changes in wavelength, hence they have no *color discrimination*
    - for example a unit amount of λ_1 versus λ_2 above
    - or a unit amount of λ_1 versus half as much of λ_3 (assuming the sensitivity to λ_3 is twice the response to λ_1)
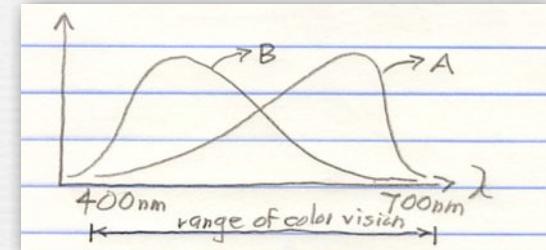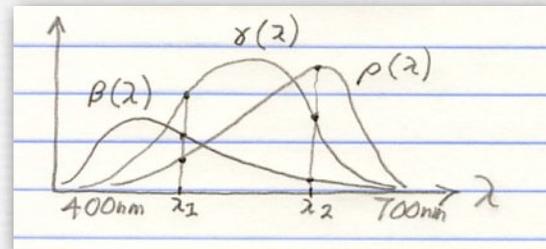
Dichromats
(contents of whiteboard)
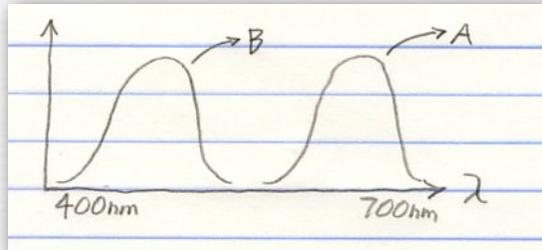
2
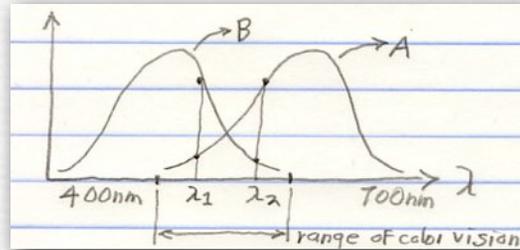
3

4

2. this organism can discrimate a response in the range wavelengths covered by A versus B, but cannot discriminate within those ranges

3. this organism has color discrimination over the range of wavelengths shown
   - for each wavelength within this range, the ratio of responses of receptors A and B is unique; hence the organism can identify which wavelength (e.g. λ_1 or λ_2) it's looking at

4. this organism has a larger range of color vision

Trichromats
(contents of whiteboard)

5

5. humans can discrimate wavelengths from 400nm to 700nm
   - we can also discriminate mixtures of wavelengths that dichromats cannot; this will become clearer later

✦ at the retinal level, our response to light is linear

    a. if the response to a unit stimulus at λ_1 of is (ρ_1, γ_1, β_1), and to a unit stimulus at λ_2 is (ρ_2, γ_2, β_2), then the response to a superposition of stimuli λ_1 and λ_2 is ($\rho_1 + \rho_2$, $\gamma_1 + \gamma_2$, $\beta_1 + \beta_2$)

    b. the response to n units of a stimulus at λ_1 is ($n\,\rho_1$, $n\,\gamma_1$, $n\,\beta_1$)

    c. a system that obeys *superposition* (a) and *scaling* (b) is *linear*

Human response to an arbitrary stimulus

spectrum of stimulus arriving
in one small area on retina

×

spectral sensitivity of each
type of cone (L,M,S)

=

multiply wavelength-by-
wavelength to get response spectra

∫

integrate over wavelengths to get
total response for that type of cone

ρ   γ   β
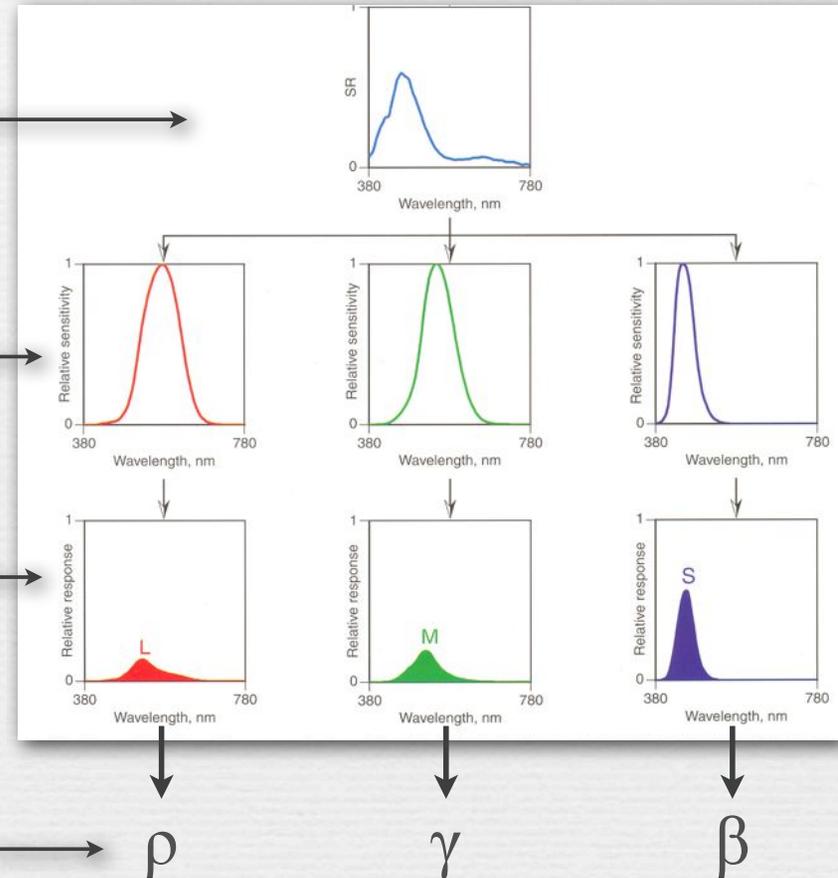
✦ output is three numbers (ρ, γ, β) per area on retina

Human response to an arbitrary stimulus
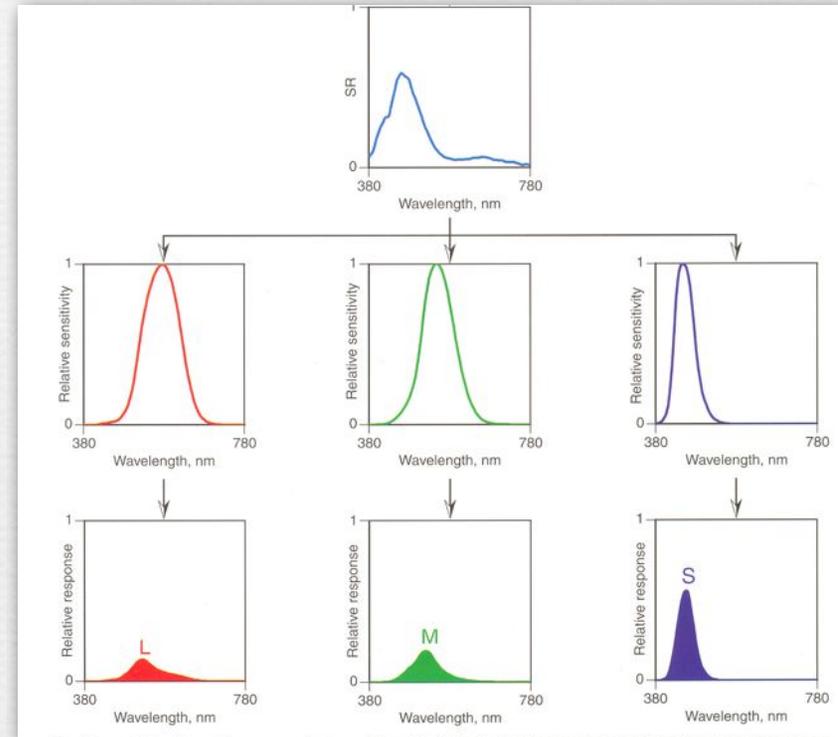
✦ stated another way, given a
  stimulus spectrum $L_e(\lambda)$, the
  human response to it (ρ, γ, β)
  are the integrals over all
  visible wavelengths of our
  responses
  $L_e(\lambda)\,\rho(\lambda),$
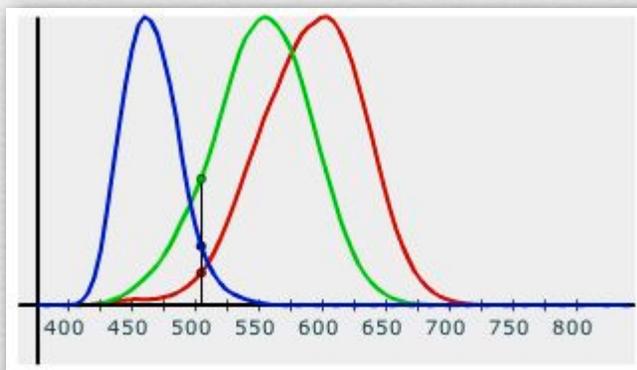 $L_e(\lambda)\,\gamma(\lambda),$
  $L_e(\lambda)\,\beta(\lambda)$
  to each constituent
  wavelength λ, i.e.

$$(\rho, \gamma, \beta) = \left(\int\limits_{400\,nm}^{700\,nm} L_e(\lambda)\,\rho(\lambda)\,d\lambda, \int\limits_{400\,nm}^{700\,nm} L_e(\lambda)\,\gamma(\lambda)\,d\lambda, \int\limits_{400\,nm}^{700\,nm} L_e(\lambda)\,\beta(\lambda)\,d\lambda \right)$$
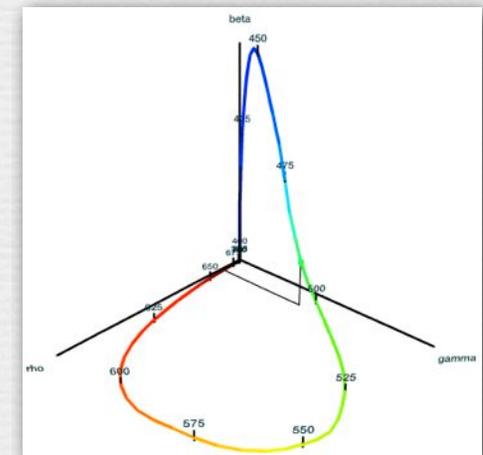
Human 3D colorspace

✦ the three types of cones in our retina (Long, Medium, Short wavelength) define the axes of a three-dimensional space

✦ our response to any stimulus spectrum can be summarized by three numbers (ρ, γ, β) and plotted as a point in this space

✦ our responses to all visible single-wavelength spectra (a.k.a. pure wavelengths λ, i.e. positions along the rainbow), if connected together, form a curve in this space, called the *locus of spectral colors*; the sequence of (ρ, γ, β) numbers form the *tristimulus sensitivity functions* $\rho(\lambda)$, $\gamma(\lambda)$, and $\beta(\lambda)$

(FLASH DEMO)

http://graphics.stanford.edu/courses/
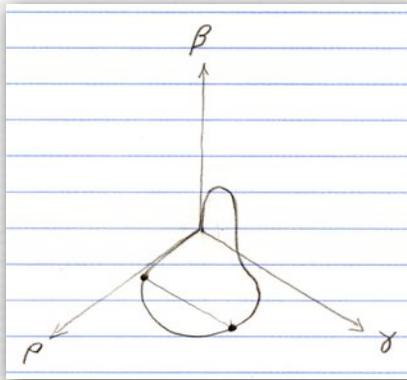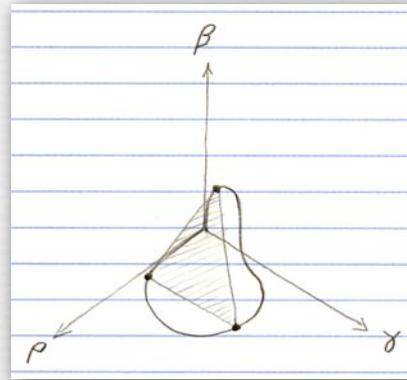cs178/applets/locus.html

sensitivity functions

spectral locus
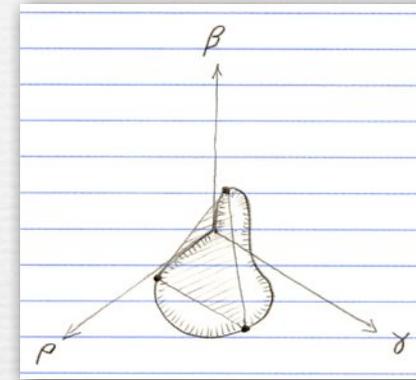
Properties of human 3D colorspace (1 of 2) (contents of whiteboard)


1   2   3

1. our response to any mixture ($\Sigma = 1$) of two pure wavelengths falls on a line connecting the responses to each wavelength

2. our response to any mixture ($\Sigma = 1$) of three pure wavelengths falls on a triangle connecting the responses to each wavelength; our response to any mixture ($\Sigma \leq 1$) of three pure wavelengths falls in a tetrahedron defined by this triangle and the origin

3. our responses to all possible mixtures ($\Sigma \leq 1$) of all visible wavelengths forms an irregular volume called the *gamut of perceivable colors,* equal to the convex hull of the spectral locus

# Properties of human 3D colorspace (2 of 2) (contents of whiteboard)
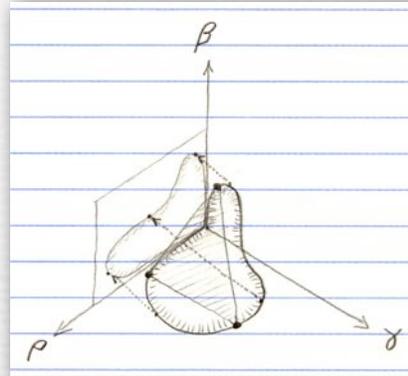
4

4. to a deuteranope - a color-blind person who is missing their medium-wavelength receptor, i.e. their gamma receptor - this diagram is squashed into the rectangle shown above on the rho-beta plane

    - as a result, spectra whose (ρ, γ, β) responses lie along the dotted lines cannot be distinguished; they will appear as the same color, i.e. as metamers

    - by a similar argument, many spectra distinguishable to pentachromats (e.g. Mallard ducks) are indistinguishable to trichromats (humans)
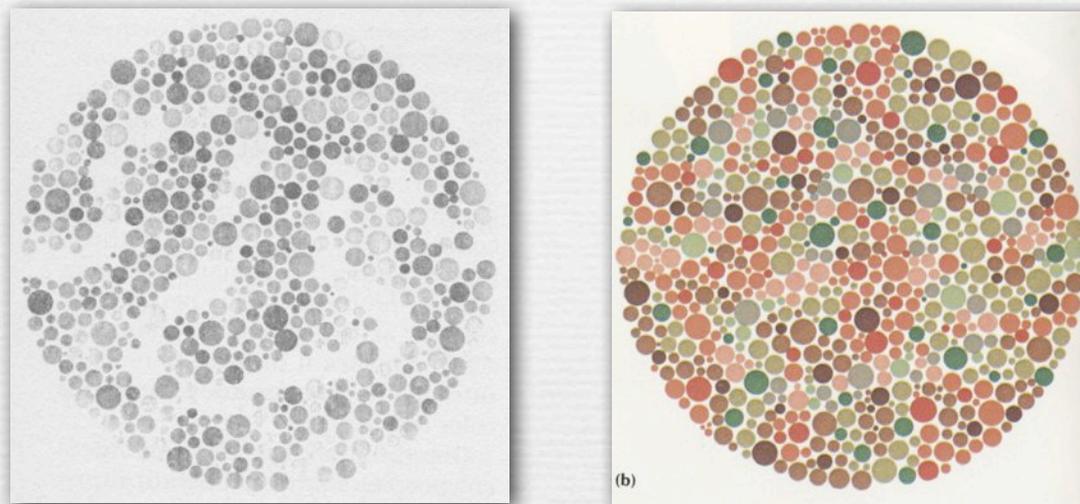
Color blindness
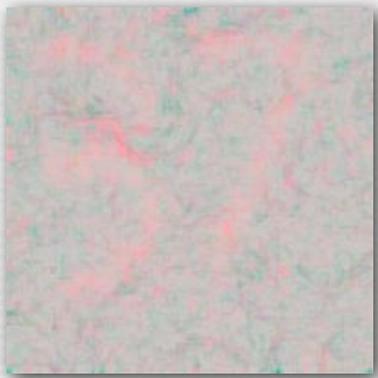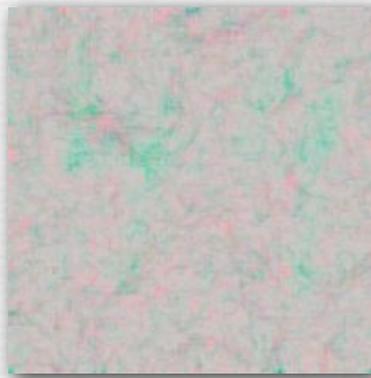
37?   49?   56?

(wikipedia)

✦ protanopia (1% of males)   ✦ protanomaly (1% of males)

✦ deuteranopia (1% of males)   ✦ deuteranomaly (6% of males)

✦ tritanopia (< 1% of both genders)   ✦ tritanomaly (< 1% of both genders)

The advantage of being color blind

(b)

✦ the maze (at left) is recreated (at right) using subtle intensity differences, but overridden by stronger red-green color differences

✦ only a deuteranope can see the maze at right

Human 3D colorspace

✦ the three types of cones in our retina (Long, Medium, Short wavelength) define the axes of a three-dimensional space

✦ our response to any stimulus spectrum can be summarized by three numbers (ρ, γ, β) and plotted as a point in this space

✦ our responses to all visible single-wavelength spectra (a.k.a. pure wavelengths λ, i.e. positions along the rainbow), if connected together, form a curve in this space, called the *locus of spectral colors*; the sequence of (ρ, γ, β) numbers form the *tristimulus sensitivity functions* ρ(λ), γ(λ), and β(λ)
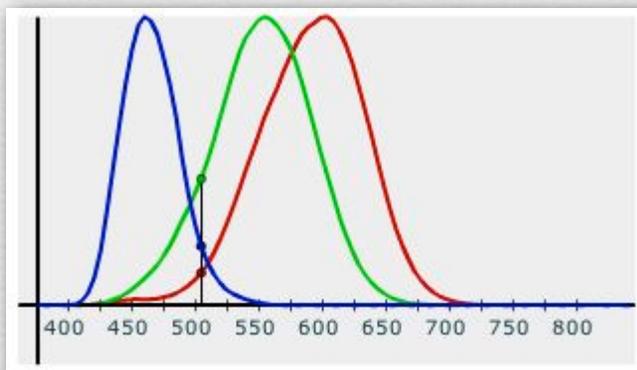
sensitivity functions

(FLASH DEMO)

http://graphics.stanford.edu/courses/
cs178/applets/locus.html
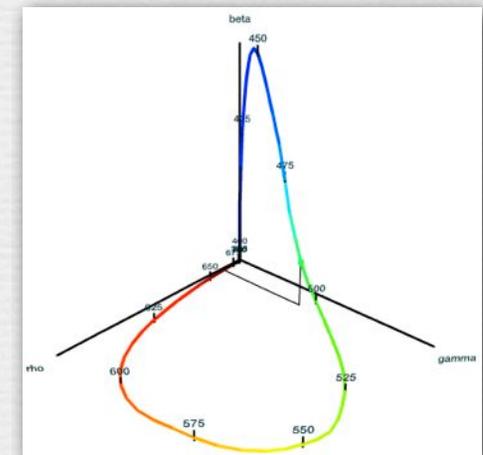
spectral locus

Maxwell's color matching experiment

✦ Maxwell actually used a slightly different procedure
  - see http://www.handprint.com/HP/WCL/color6.html for details
  - the procedure below is used in modern versions of the experiment

(FLASH DEMO)

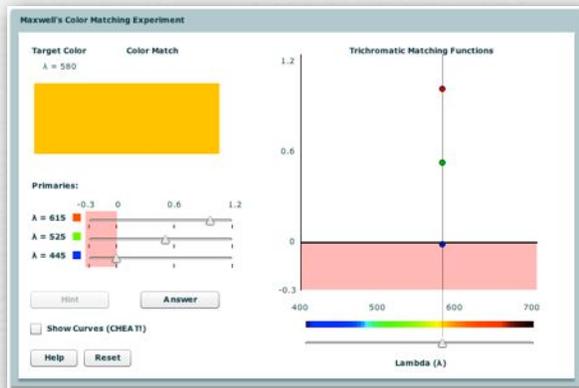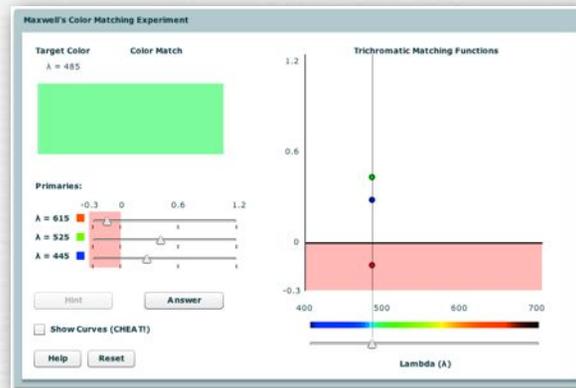http://graphics.stanford.edu/courses/
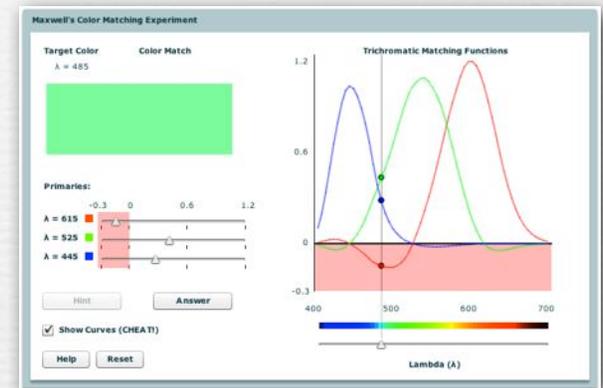cs178/applets/colormatching.html
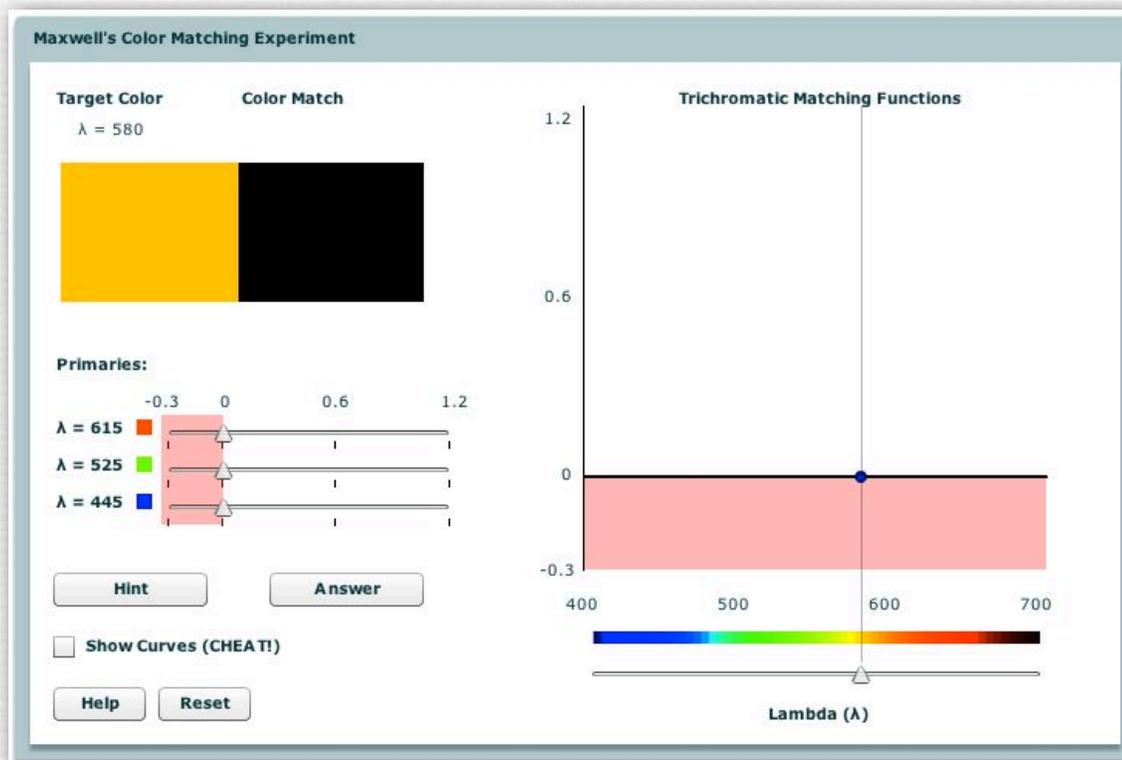
Maxwell's color matching experiment
(summary of live demo)

1.   given a stimulus wavelength, the amount of each primary required to match it is given by three numbers (r, g, b)

2.   some stimuli cannot be matched unless first desaturated by adding a primary to it before matching; the amount added is denoted by negative values of $r, g,$ or b

3.   the sequence of $(\bar{r}, \bar{g}, \bar{b})$ values, some negative, required to match the locus of spectral colors across all λ, form the *trichromatic matching functions* $\bar{r}(\lambda), \bar{g}(\lambda),$ and $\bar{b}(\lambda)$ for a particular set of 3 primaries

Young-Helmholtz trichromatic theory

Thomas Young
(1773-1829)

James Clerk Maxwell
(c. 1860)

Hermann von Helmholtz
(1821-1894)

✦ spectra can be visually matched using mixtures of *primary colors*; such matches are called *metamers*

✦ due to the <u>linearity</u> of human retinal response, given a stimulus spectrum $L_e(\lambda)$, the amounts of each primary R, G, B required to match it, for any particular choice of 3 primaries, are the integrals over all visible wavelengths of the amounts $\bar{r}(\lambda)$, $\bar{g}(\lambda)$, and $\bar{b}(\lambda)$ required to match each constituent wavelength λ, *i.e.*

$$(R,G,B) = \left(\int_{400\,nm}^{700\,nm} L_e(\lambda)\,\bar{r}(\lambda)\,d\lambda,\ \int_{400\,nm}^{700\,nm} L_e(\lambda)\,\bar{g}(\lambda)\,d\lambda,\ \int_{400\,nm}^{700\,nm} L_e(\lambda)\,\bar{b}(\lambda)\,d\lambda \right)$$

3D interpretation of color matching

✦ our response to varying amounts of a primary forms a vector in (ρ, γ, β) space, rooted at the origin

✦ to provide a normal range of color vision, three primaries are required, and their vectors must not lie on a plane

✦ our responses to all possible mixtures ($\Sigma \leq 1$) of three primaries form a tetrahedron called the *gamut of reproducible colors* for these primaries
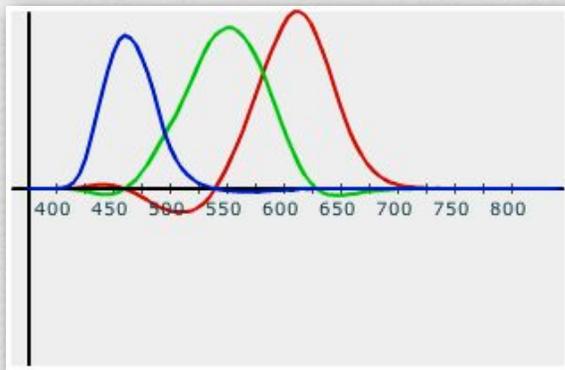

RGB matching functions

(FLASH DEMO)

http://graphics.stanford.edu/courses/cs178/applets/locus.html
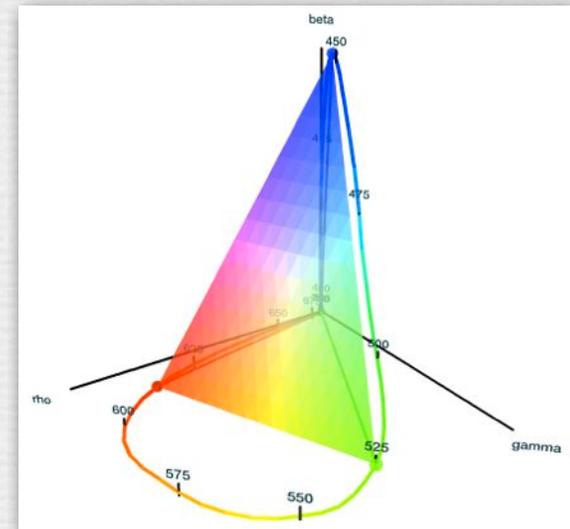
gamut of reproducible colors

3D interpretation of color matching

- ✦ the spectrum of each of the three primaries can be a pure wavelength (1) or a mixture of wavelengths (2)

- ✦ impure primaries have a smaller gamut in (ρ, γ, β) space

- ✦ additional primaries can be added to increase the gamut

1

2

(FLASH DEMO)

http://graphics.stanford.edu/courses/cs178/applets/locus.html
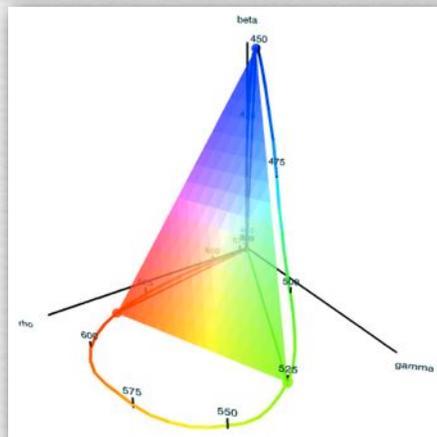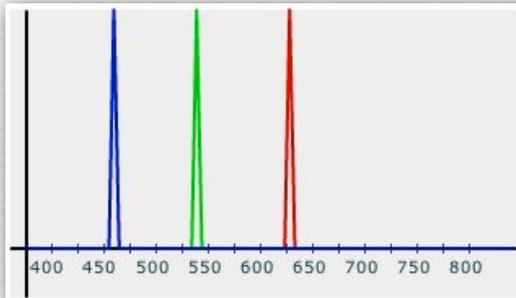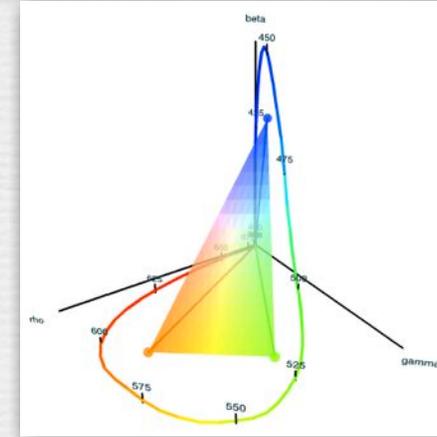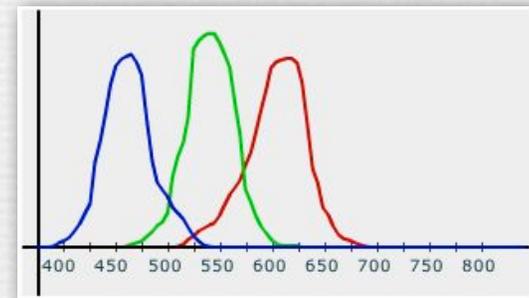
Chromaticity diagrams

✦ choose three primaries R,G,B, pure wavelengths or not

✦ adjust R=1,G=1,B=1 to obtain a desired *reference white*

✦ this yields an *RGB cube*

(FLASH DEMO)
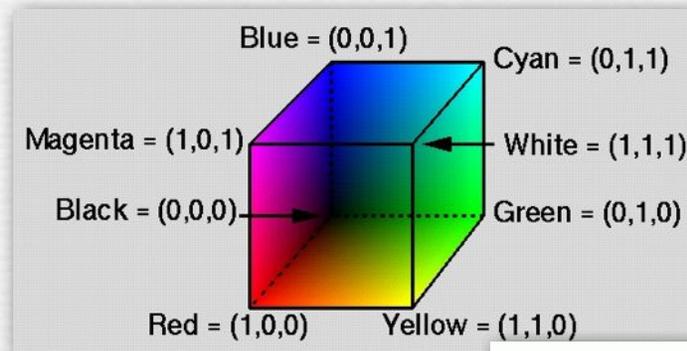
$$r = \frac{R}{R+G+B}$$

$$g = \frac{G}{R+G+B}$$

✦ one may factor the brightness out of any point in the cube by drawing a line to the origin and intersecting this line with the triangle made by corners Red, Green, Blue

✦ all points on this triangle, which are addressable by two coordinates, have the same brightness but differing *chromaticity*

19

©Marc Levoy
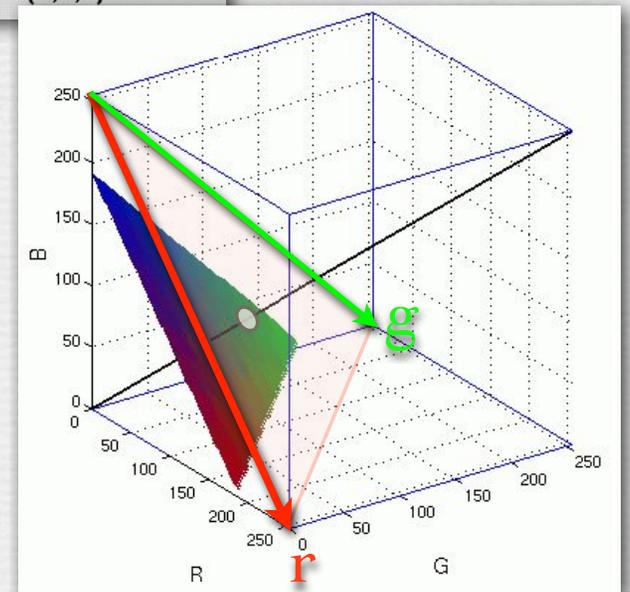
Chromaticity diagrams

- ✦ this triangle is called the *rgb chromaticity diagram* for the chosen RGB primaries
  - mixtures of colors lie along straight lines
  - neutral (black to white) lies at (⅓, ⅓)
  - r>0, g>0 does not enclose spectral locus

- ✦ the same construction can be performed using <u>any</u> set of 3 vectors as primaries, even impossible ones ($\rho < 0$ or $\gamma < 0$ or $\beta < 0$)

- ✦ the CIE has defined a set of primaries XYZ, and the associated *xyz chromaticity diagram*
  - x>0, y>0 <u>does</u> enclose spectral locus
  - one can connect red and green on the locus with a *line of extra-spectral purples*
  - x,y is a standardized way to denote colors

rgb chromaticity diagram

CIE xyz chromaticity diagram

(Hunt)
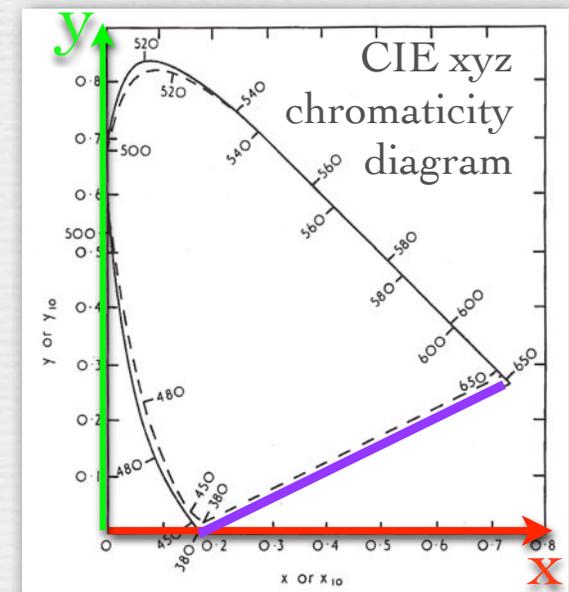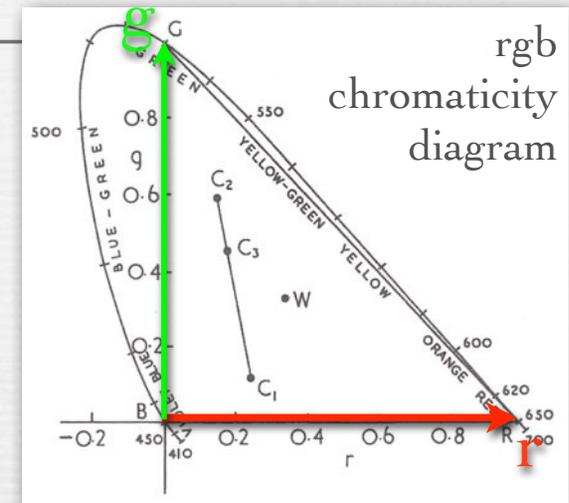
Example pigment catalog

http://www.webexhibits.org/
pigments/intro/pigments.html

© Marc Levoy

XYZ values for Prussian Blue
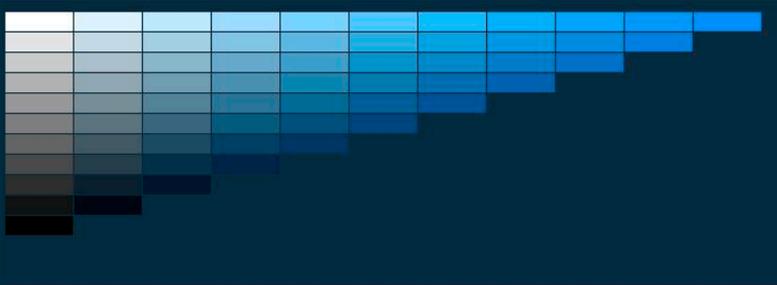
# Application of chromaticity diagrams #1: standardized color spaces and gamut mapping

✦ the chromaticities reproducible using 3 primaries fill a triangle in the xyz chromaticity diagram, a different triangle for each choice of primaries; this is called the *device gamut* for those primaries

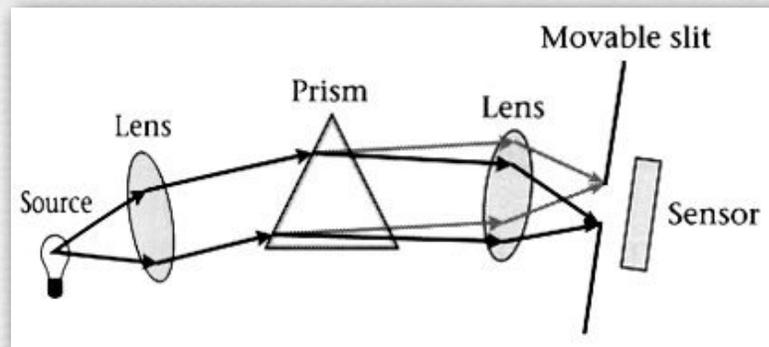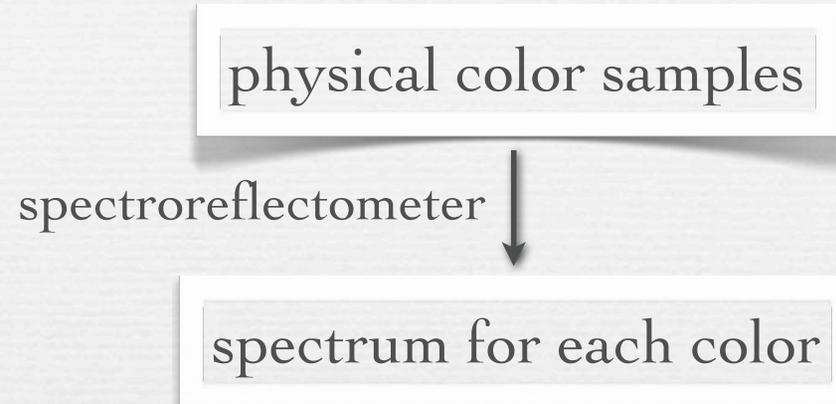Q. Why is this diagram, scanned from a book, black outside the printer gamut?
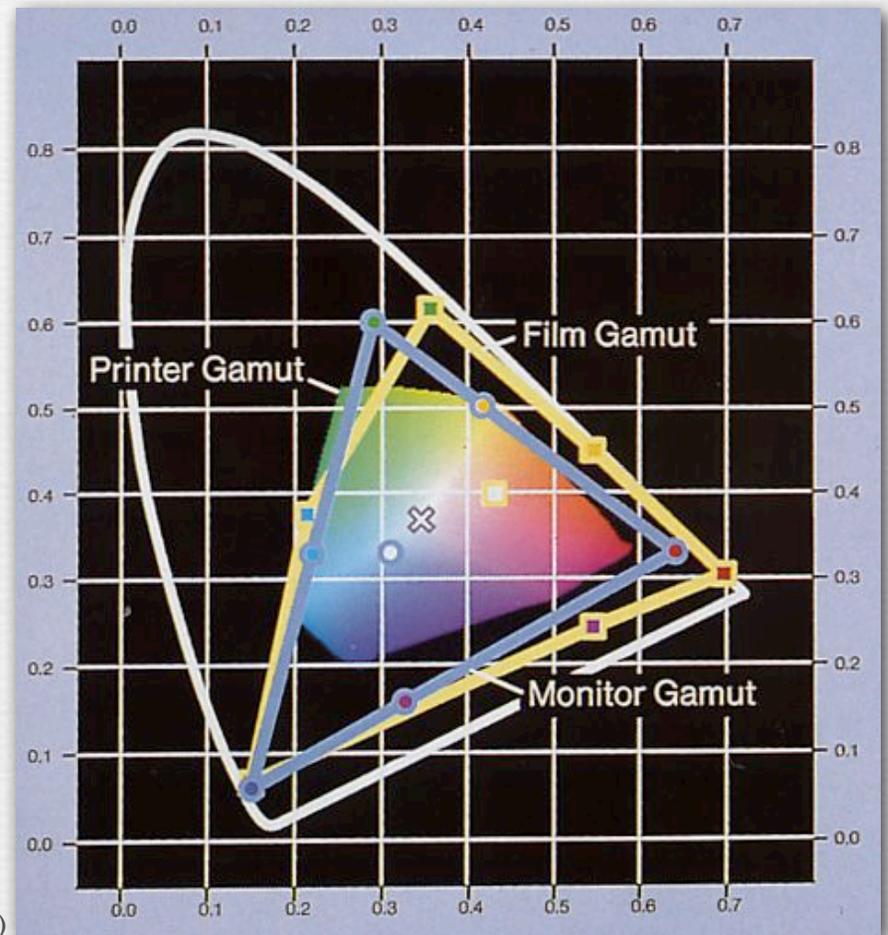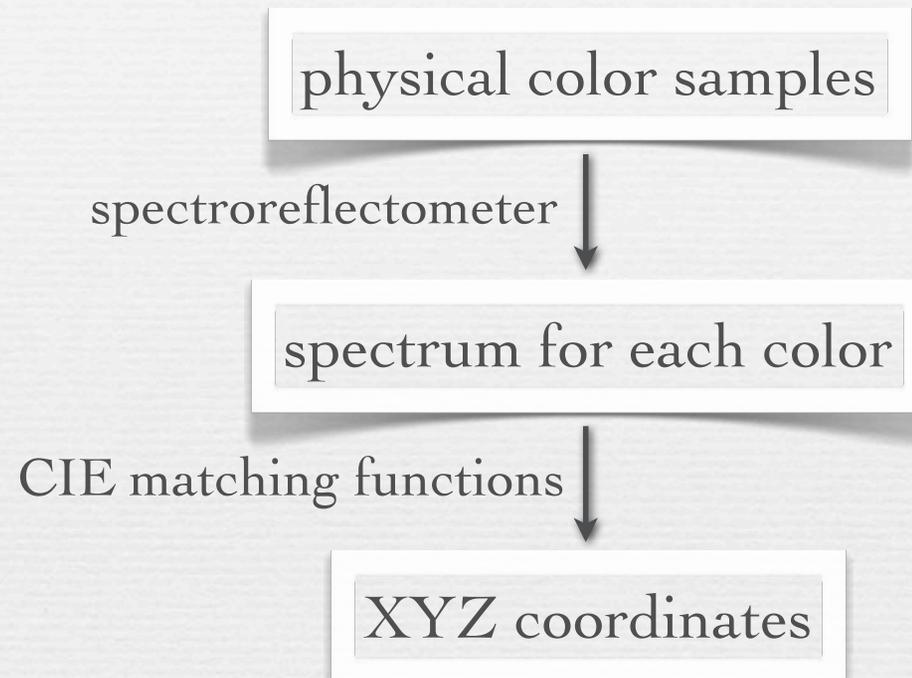
(Foley)
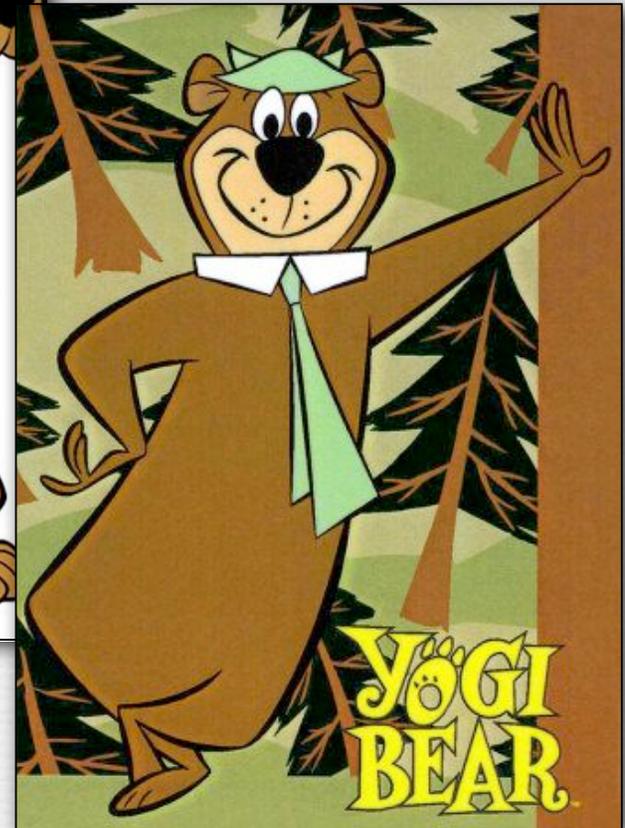
Digitizing the paint colors at Hanna-Barbera Productions
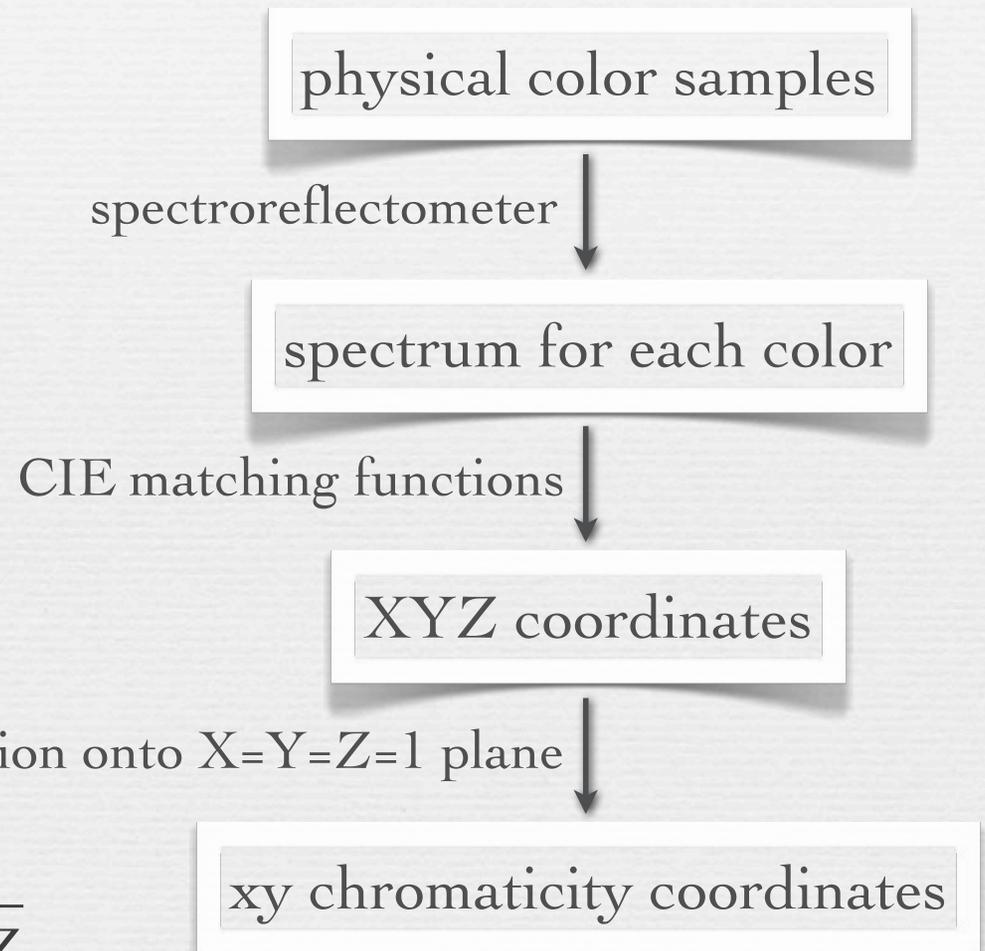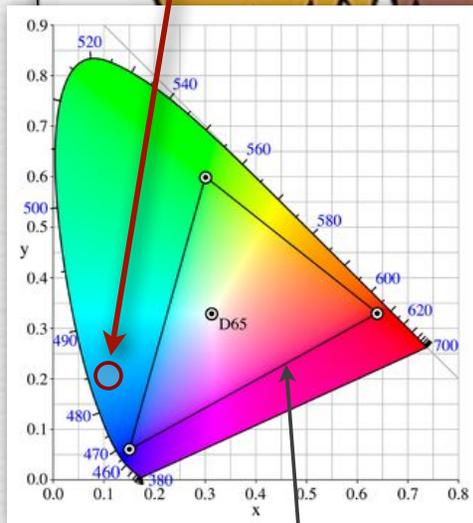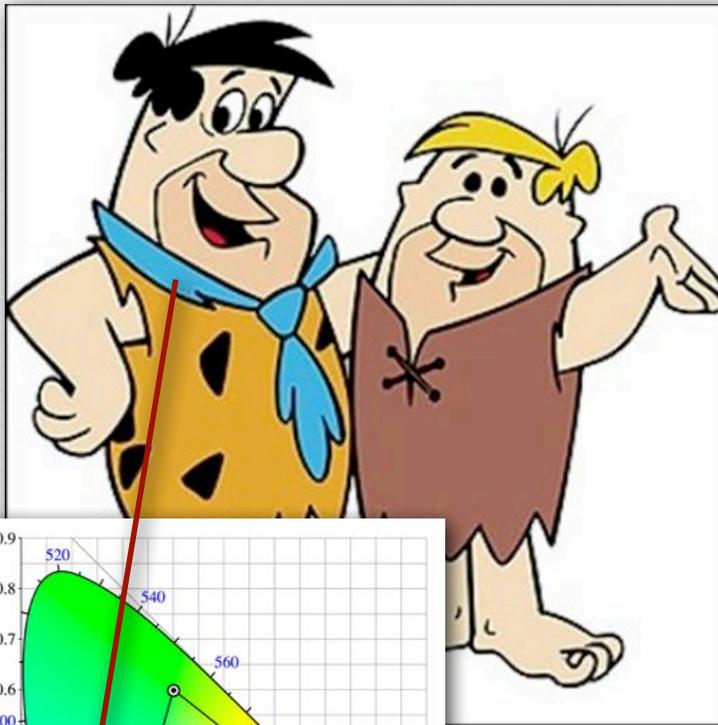
Digitizing the paint colors at Hanna-Barbera Productions

physical color samples

spectroreflectometer

spectrum for each color

Digitizing the paint colors at Hanna-Barbera Productions

physical color samples

spectroreflectometer

spectrum for each color

CIE matching functions

XYZ coordinates

$$(X,Y,Z) = \left(\int_{400\,nm}^{700\,nm} L_e(\lambda)\,\bar{x}(\lambda)\,d\lambda, \; \int_{400\,nm}^{700\,nm} L_e(\lambda)\,\bar{y}(\lambda)\,d\lambda, \; \int_{400\,nm}^{700\,nm} L_e(\lambda)\,\bar{z}(\lambda)\,d\lambda \right)$$
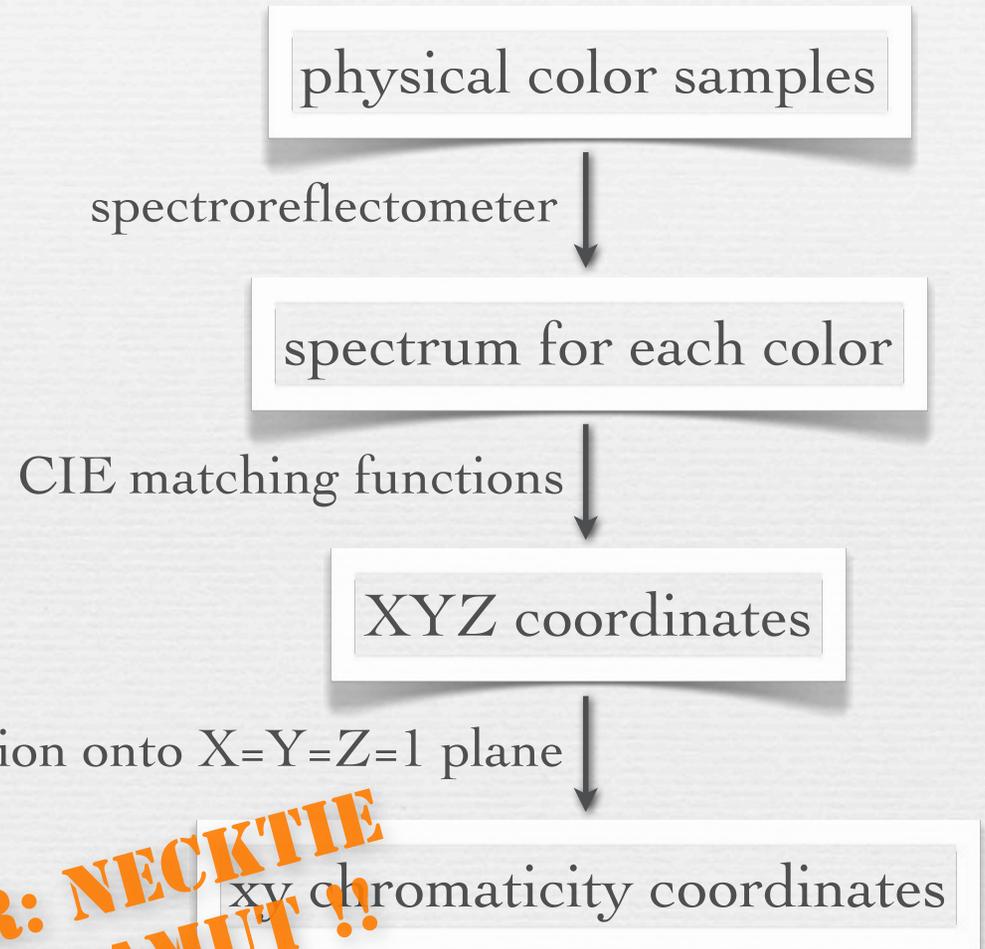
Digitizing the paint colors at Hanna-Barbera Productions

physical color samples

spectroreflectometer ↓

spectrum for each color

CIE matching functions ↓

XYZ coordinates

projection onto X=Y=Z=1 plane ↓

xy chromaticity coordinates

$$x = \frac{X}{X+Y+Z} \qquad y = \frac{Y}{X+Y+Z}$$

Digitizing the paint colors at Hanna-Barbera Productions

physical color samples

spectroreflectometer

spectrum for each color

CIE matching functions

XYZ coordinates

projection onto X=Y=Z=1 plane

xy chromaticity coordinates

DANGER: NECKTIE OUT OF GAMUT !!

NTSC gamut

Gamut mapping

✦ called "color space conversion" in Photoshop

- *relative colorimetric* - shrinks only out-of-gamut colors, towards N
- *absolute colorimetric* - same but shrinks to nearest point on gamut
- *perceptual* - smoothly shrinks all colors to fit in target gamut
- *saturated* - sacrifices smoothness to maintain saturated colors

(FLASH DEMO)

http://graphics.stanford.edu/courses/cs178/applets/gamutmapping.html

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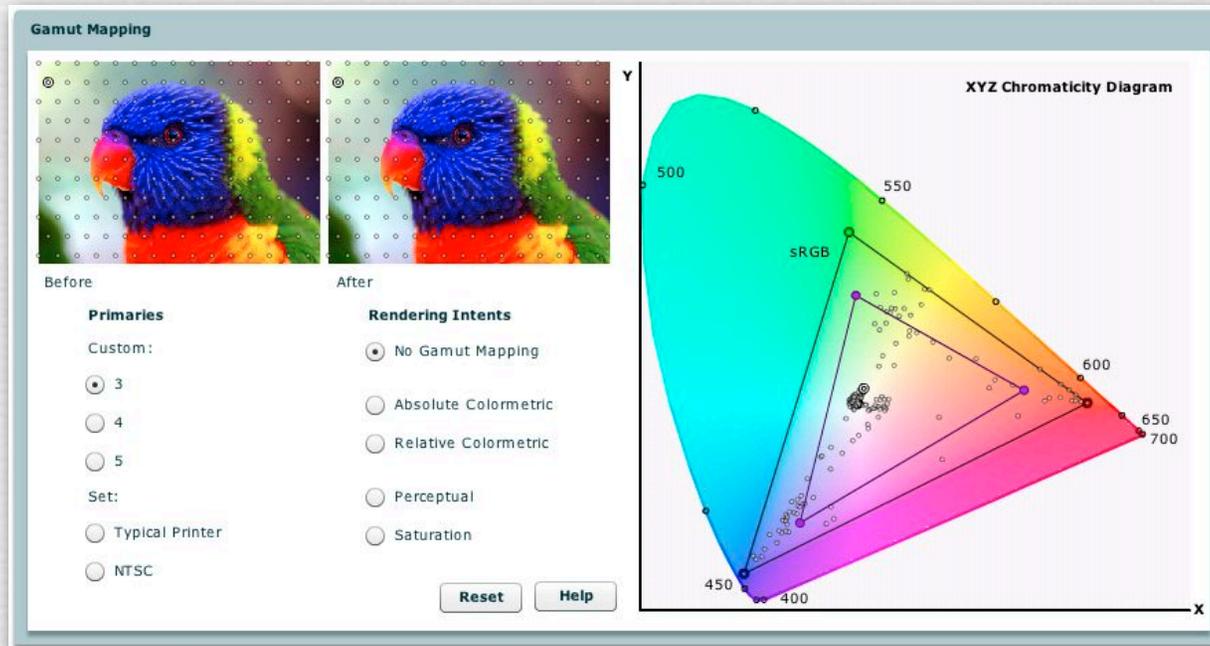
Application of chromaticity diagrams #2: color temperature and white balancing
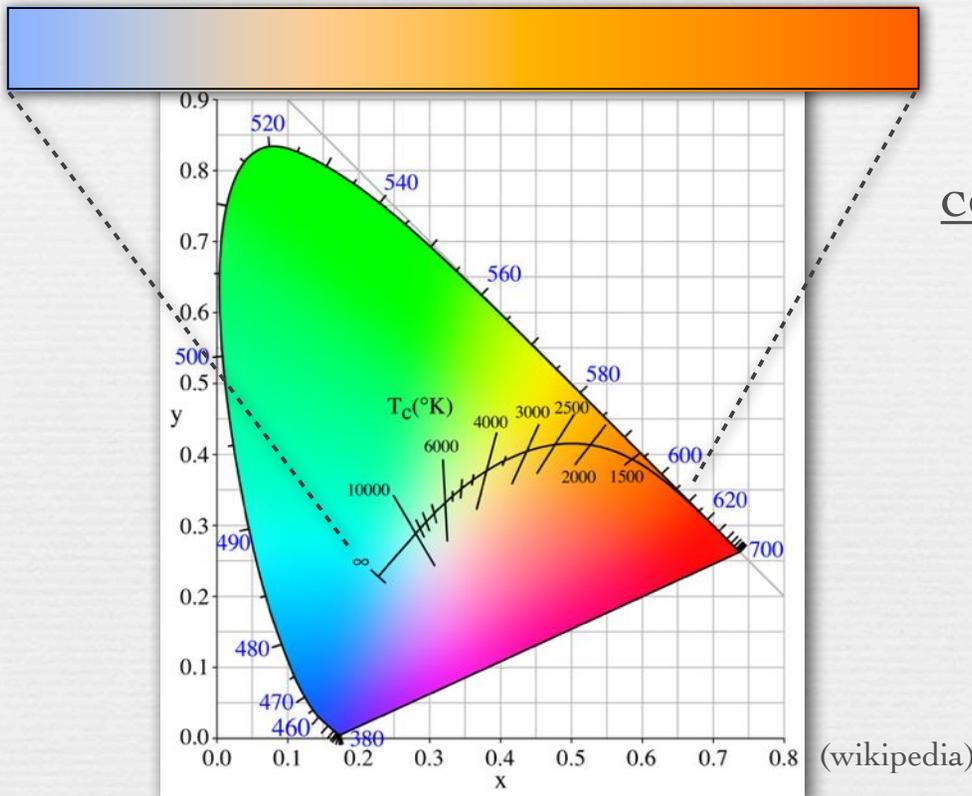
correlated color temperatures

3200°K incandescent light

4000°K cool white fluorescent

5000°K equal energy white (D50, E)

6000°K midday sun, photo flash

6500°K overcast, television (D65)

7500°K northern blue sky

(wikipedia)

✦ the apparent colors emitted by a *black-body radiator* heated to different temperatures fall on a curve in the chromaticity diagram

✦ for non-blackbody sources, the nearest point on the curve is called the *correlated color temperature*
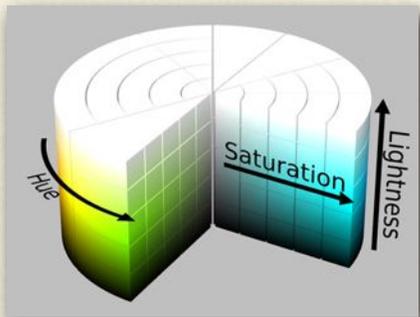
© Marc Levoy
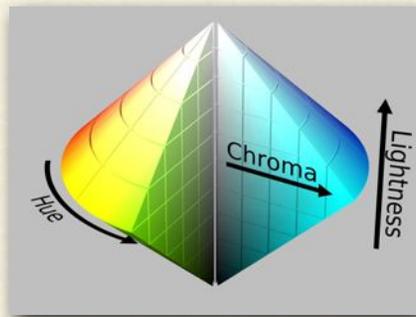
Incorrectly chosen white balance

(Eddy Talvala)

✦ scene was photographed in sunlight, but the camera's white balance was set as if the illumination were "warmer", like tungsten
  • white balancer assumed illumination was reddish, so it boosted blues
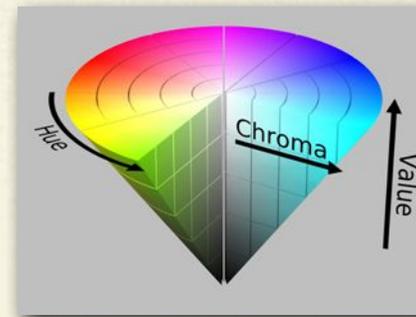
# Cylindrical color spaces

HSL cylinder   HSL double cone   HSV single cone

- ❖ a cylinder is easy to understand, but colors near the top and bottom are indistinguishable

  - ✴ single cone solves this by compressing top & bottom to a point

- ❖ when artists mix "complementary" lights, they expect to get white, but halfway from red to cyan in HSL space is gray

  - ✴ HSV pushes the white point down onto the max-S plane

  - ✴ painters might prefer an inverted cone, with black on this plane

Munsell color system
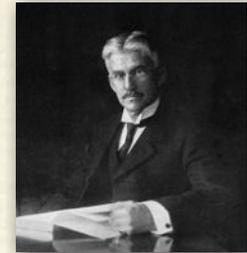
Albert Munsell
(1858-1918)
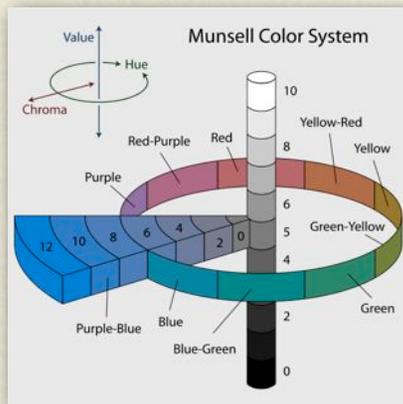
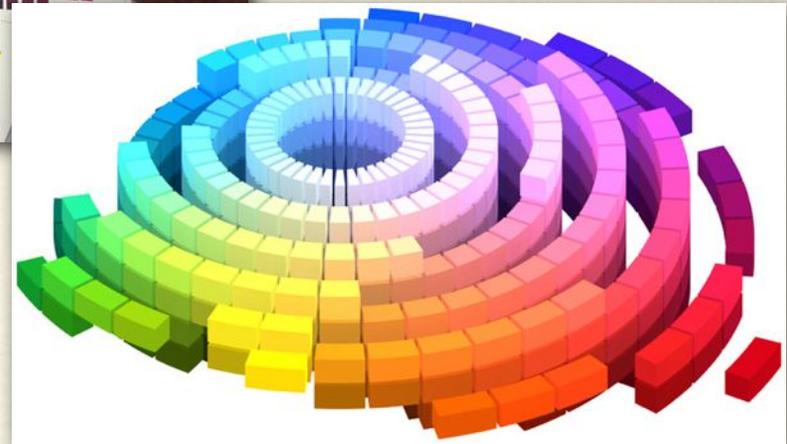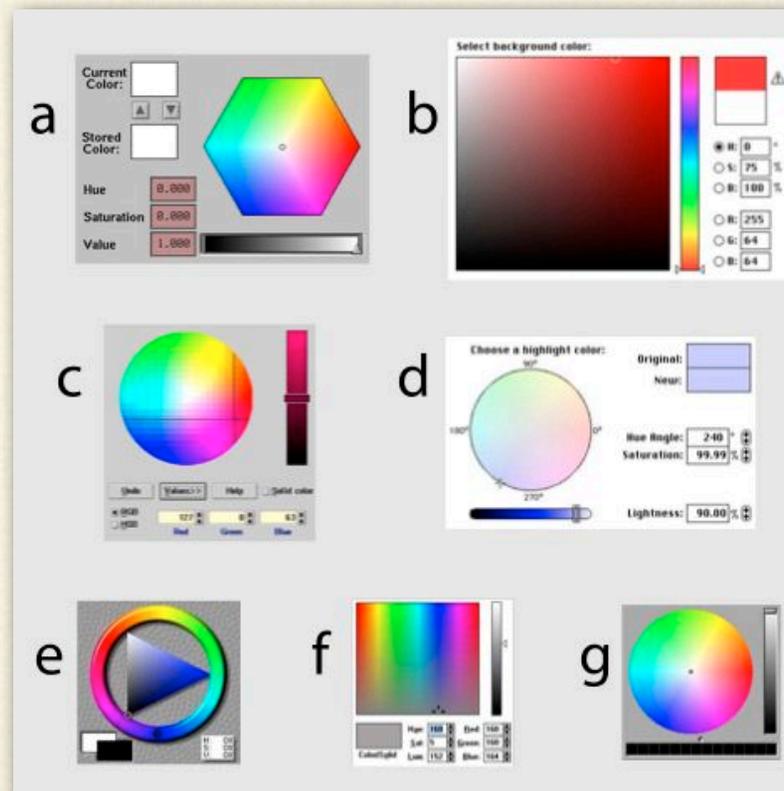3-axis colorspace

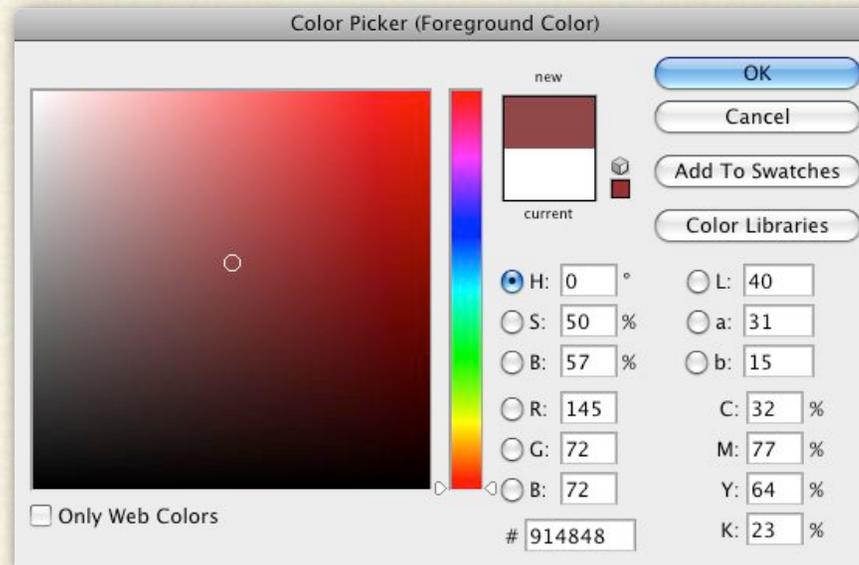1905 book

CG rendering of
1929 measurements

❖ spacing of colors is perceptually uniform (by experiment)

❖ outer envelope of solid determined by available inks
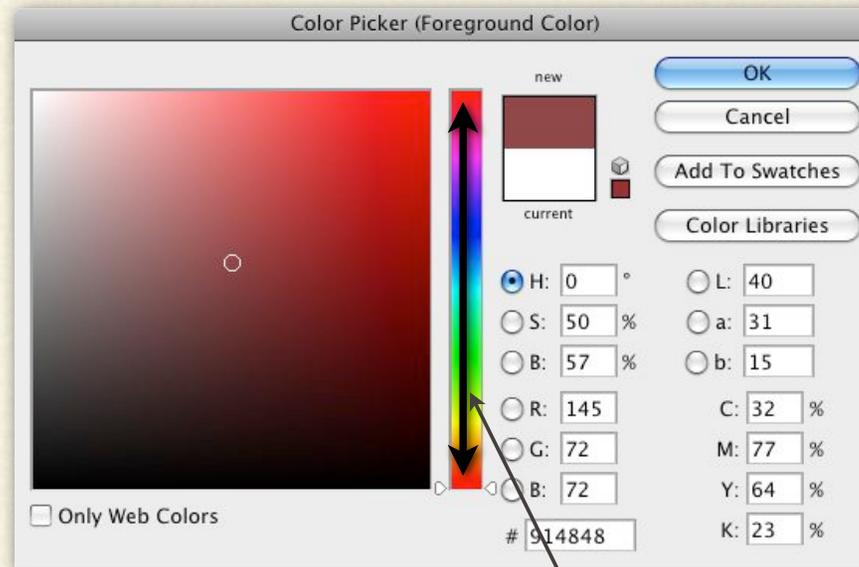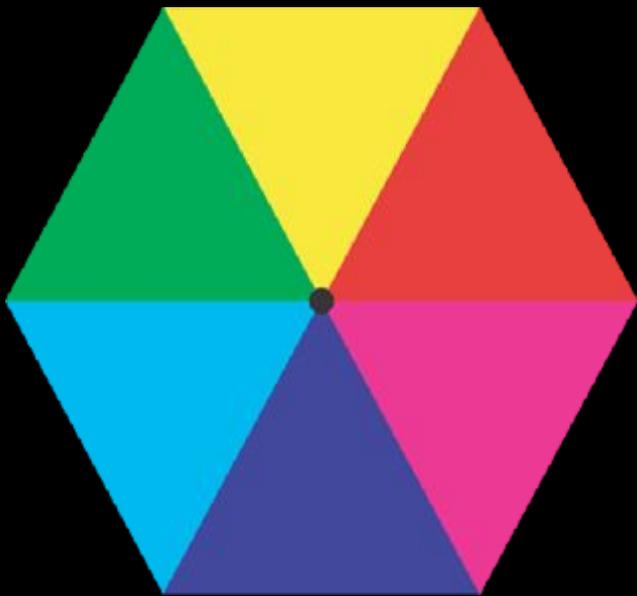
A menagerie of color selectors

Color selection in Photoshop
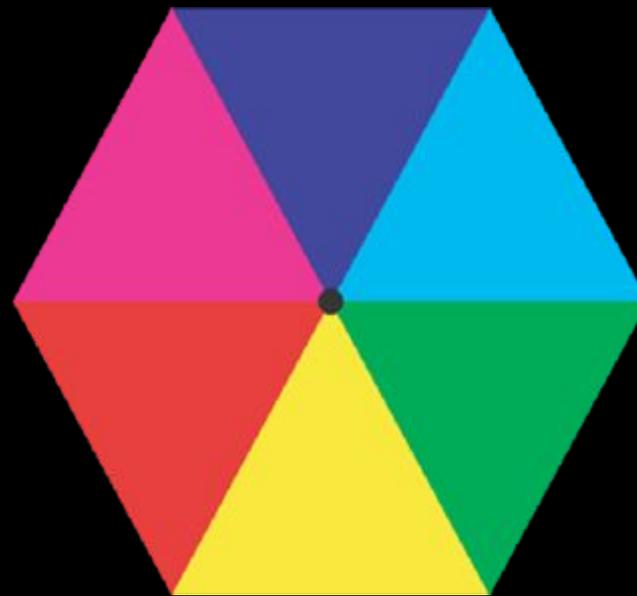
Color selection in Photoshop

brightness

saturation   hue

image

afterimage
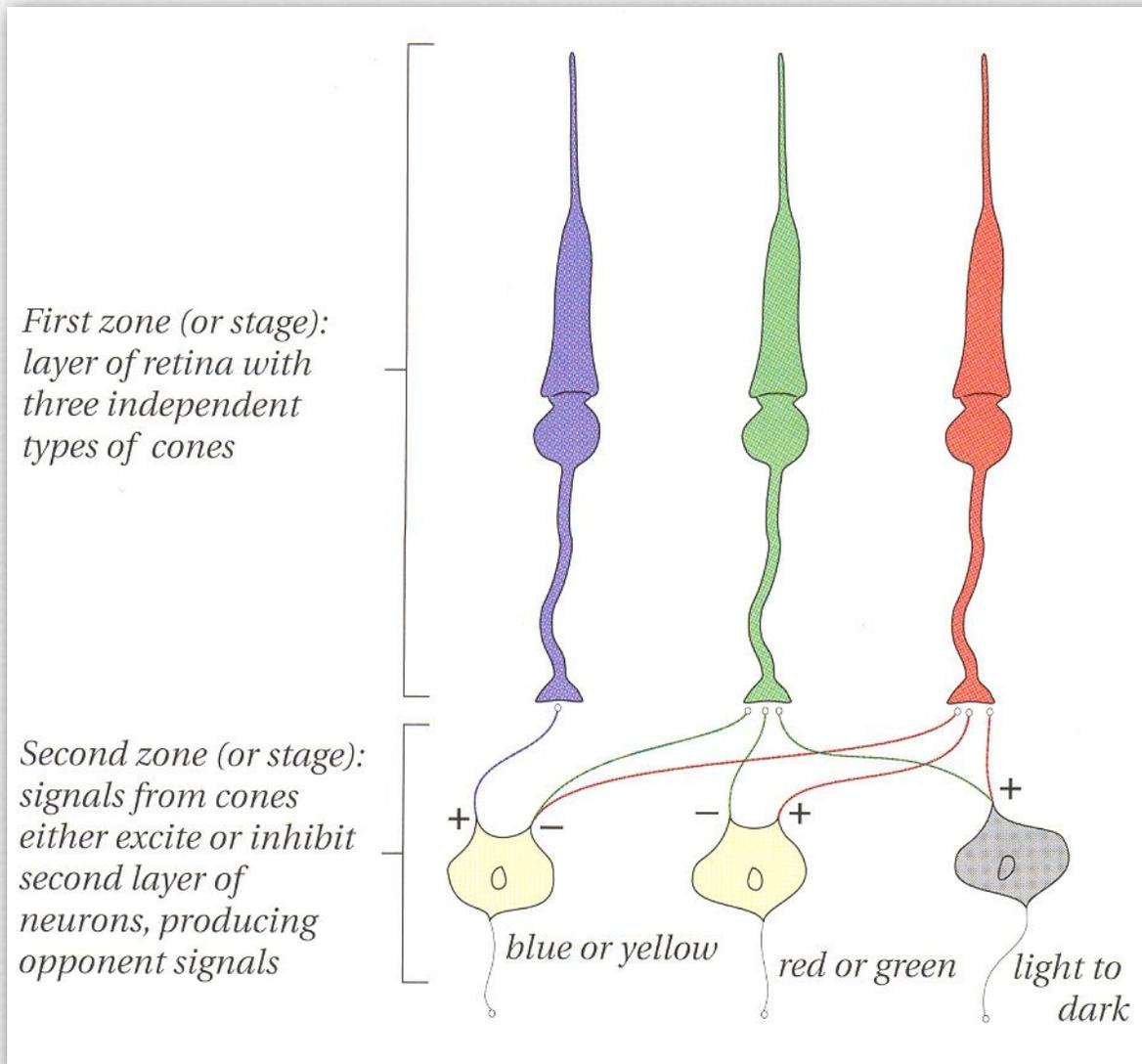
Opponent colors

Ewald Hering
(1834-1918)

red/green
receptors

blue/yellow
receptors

black/white
receptors
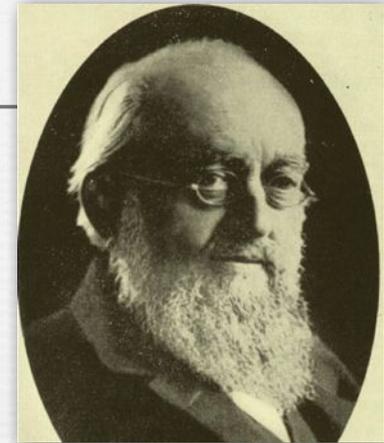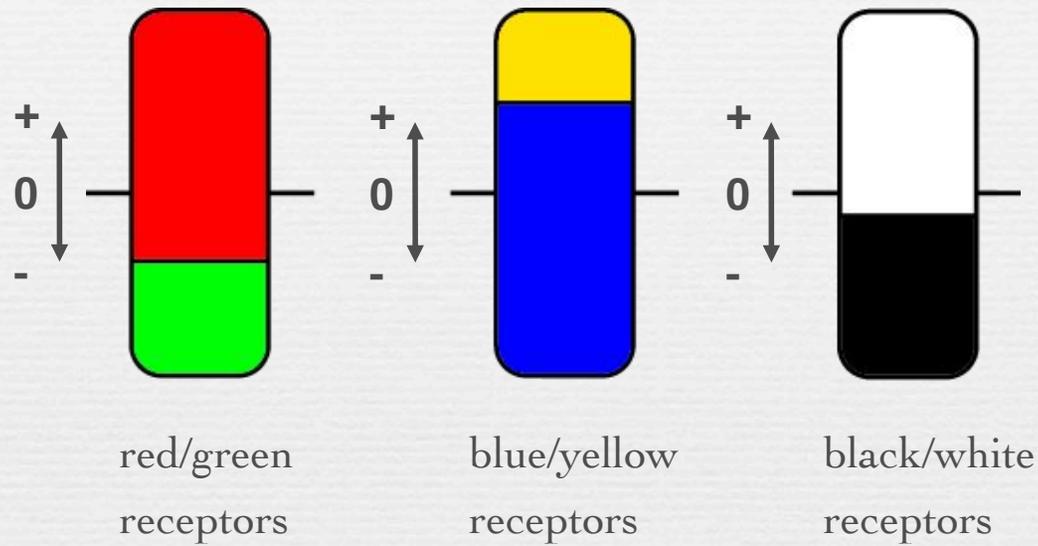
✦ observed that humans don't see reddish-green colors or blueish-yellow colors

✦ hypothesized three receptors, as shown above

Opponent colors wiring

First zone (or stage): layer of retina with three independent types of cones

Second zone (or stage): signals from cones either excite or inhibit second layer of neurons, producing opponent signals

blue or yellow   red or green   light to dark
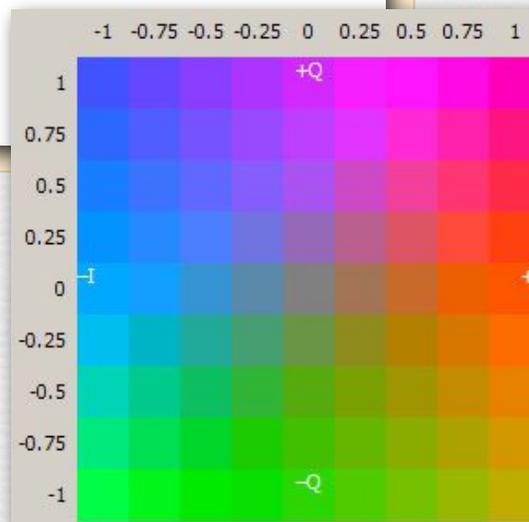
Practical use of opponent colors: NTSC color television

✦ color space is YIQ
- Y = luminance
- I = orange-blue axis
- Q = purple-green axis

$$R, G, B, Y \in [0,1], \quad I \in [-0.5957, 0.5957], \quad Q \in [-0.5226, 0.5226]$$

$$\begin{bmatrix} Y \\ I \\ Q \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ 0.595716 & -0.274453 & -0.321263 \\ 0.211456 & -0.522591 & 0.311135 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 1 & 0.9563 & 0.6210 \\ 1 & -0.2721 & -0.6474 \\ 1 & -1.1070 & +1.7046 \end{bmatrix} \begin{bmatrix} Y \\ I \\ Q \end{bmatrix}$$
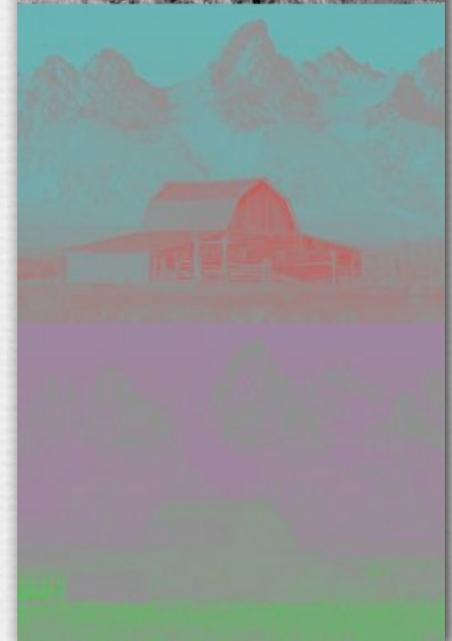
RGB & YIQ are axes in (ρ, γ, β) space, hence these transforms are 3×3 matrix multiplications
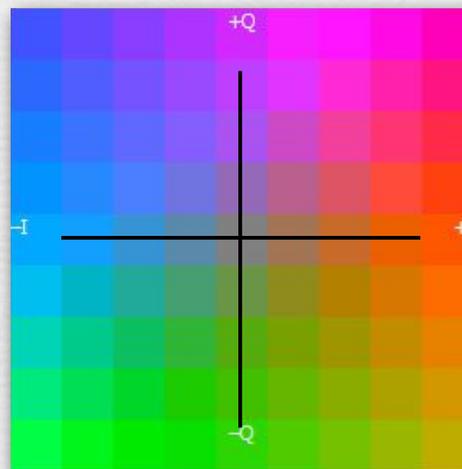
Y

I

Q

© Marc Levoy

Practical use of opponent colors: JPEG image compression

✦ color space is Y'CbCr

- Y' = luminance
- Cb = yellow-blue axis
- Cr = red-green axis

$$Y' = 16 + (65.481 \cdot R' + 128.553 \cdot G' + 24.966 \cdot B')$$
$$C_B = 128 + (-37.797 \cdot R' - 74.203 \cdot G' + 112.0 \cdot B')$$
$$C_R = 128 + (112.0 \cdot R' - 93.786 \cdot G' - 18.214 \cdot B')$$

Y'

Cb

Cr

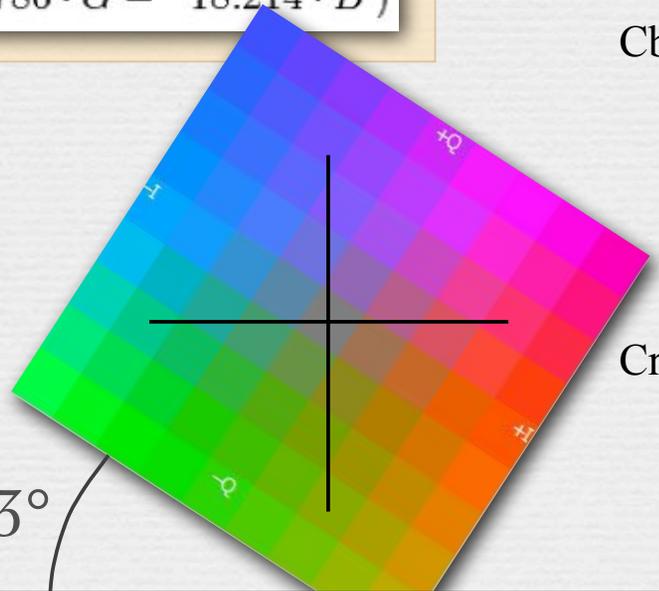# Practical use of opponent colors: JPEG compression

✦ color space is YCbCr
  - Y = luminance
  - Cb = yellow-blue axis
  - Cr = red-green axis

we are more sensitive to high frequencies in Y than CbCr, so use more bits for Y (~2×)

$$Y' = 16 + (65.481 \cdot R' + 128.553 \cdot G' + 24.966 \cdot B')$$
$$C_B = 128 + (-37.797 \cdot R' - 74.203 \cdot G' + 112.0 \cdot B')$$
$$C_R = 128 + (112.0 \cdot R' - 93.786 \cdot G' - 18.214 \cdot B')$$

inputs R', G', B' are R^γ, G^γ, B^γ for some gamma $\gamma < 1$
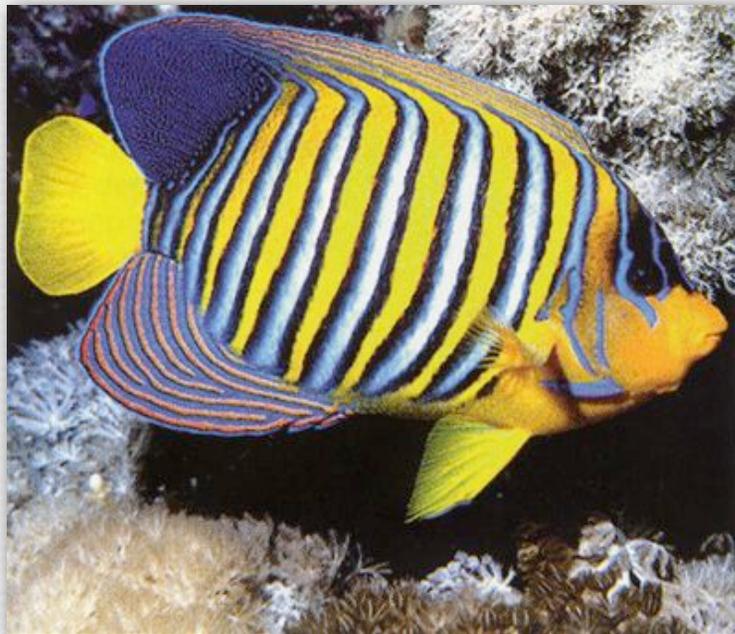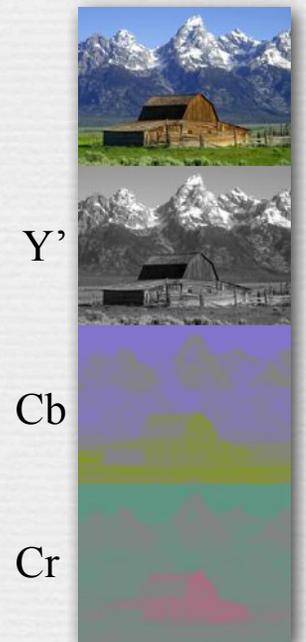
Y'

Cb

Cr

33°

Apparent spatial sharpness depends mainly on luminance, not chrominance

original image

(Wandell)

Y'

Cb

Cr

Apparent spatial sharpness depends mainly on luminance, not chrominance

(Wandell)

red-green
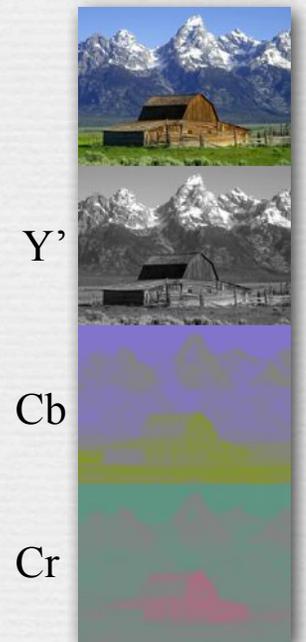channel (Cr)
blurred

Y'

Cb

Cr

Apparent spatial sharpness depends mainly on luminance, not chrominance

original
image

(Wandell)

Y'

Cb

Cr

Apparent spatial sharpness depends mainly on luminance, not chrominance

(Wandell)

blue-yellow
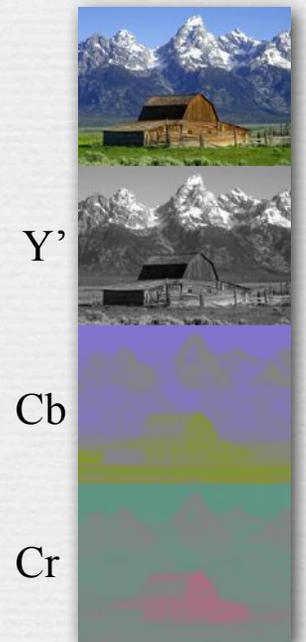channel (Cb)
blurred

Y'

Cb

Cr

Apparent spatial sharpness depends mainly on luminance, not chrominance

original
image

(Wandell)

Y'

Cb

Cr

Apparent spatial sharpness depends mainly on luminance, not chrominance

(Wandell)

luminance
channel (Y')
blurred

Y'

Cb

Cr

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