CS 468 (Spring 2013) — Discrete Differential Geometry

Course outline

1. (J) Overview of the course

2. (A) Curves
   - Define curves and related objects
   - Frenet frame and Bishop frame
   - Fundamental theorem of plane curves, turning angles

3. (J) Discrete curves
   - Choice of curvature measure
   - Turning angle etc
   - Bishop frame, rods paper
   - How to compute Frenet/Bishop frame

4+5. (A) Surface theory (embedded in $\mathbb{R}^n$)
   - Definition of surface, tangent space
   - Change of coordinates
   - Parametric vs. implicit representations Examples
   - Define what “topology” means vs. geometry; the Euler characteristic.

6. (J) Surface practice
   - Halfedge vs. implicit vs. other mesh representations
   - Surfaces via point clouds, Maya/subdivision, RBF/implicit, simulation, procedural
   - Euler characteristic

7. (A) Extrinsic curvature (second fundamental form, Gauss map, etc)

8. (J) Computing curvature.

9. (A) The intrinsic metric
   - Definition and properties
   - Length and geodesics
   - Volume and integration (w.r.t. measure)

10. (J) Geodesic computation via fast marching methods

11. (A) Derivatives
   - Definition of covariant derivative of a vector field along a curve and a surface
   - Prove relationship to second fundamental form
   - Intrinsic nature of the projected derivative; Christoffel symbols
   - Fundamental Lemma of Riemannian Geometry
   - Gradient vector field, divergence, Laplacian

12. (J) The Laplace operator of a mesh
• Cotan Laplace
• Solve the heat equation (time integration or eigenfunctions)
• HKS
• Mention that we can’t do general covariant derivatives of vector fields in the same way!

13. (A) Surface deformation
• Isometries and Gauss’ Totally Awesome Theorem
• Rigidity and the Gauss Bonnet theorem
• Fundamental theorem of Riemannian geometry of surfaces. Motivation for what’s next?
• Smoothing and curvature flows
• Bending/stretching energies and elasticity

14. (J) Surface deformation practice
• ARAP
• Time integration
• Cloth/thin shells
• Classification of bending/stretching energies and approximations thereof

* Possible section on Math 52 concepts.

15. (A) Exterior Calculus
• Div, grad, curl
• Define forms
• Stokes’ Theorem on a surface
• De Rham cohomology of a surface
• Hodge decomposition of vector fields

16. (J) Discrete Exterior Calculus
• Integration of vector fields
• Discrete Hodge decomposition
• FEM interpretation
• Relationship to cotan Laplacian

17. (A) PDEs on surfaces
• Laplace equation, heat equation, wave equation, Schrödinger equation
• Some analytical considerations e.g. coercivity of Laplace’s equation

18. (J) Numerical solution of PDEs on meshed surfaces
• Application papers and techniques
• FEM approach
• Integration, stability, convergence
• Barycentric coordinates

19. (A+J) Conformal geometry
• Continuous and discrete conformal geometry
• Parameterization
• Mapping