Vectors, dual vectors and tensors.
- Let $V$ be a real vector space with an inner product $g$.
- The dual space $V^*$ and the sharp and flat opertors. Induced inner product.
- What happens when we choose a basis. The dual basis. Orthonormality.
- The tensor products $V^* \otimes \cdots \otimes V^*$. Induced inner product.
- Mixed tensors. The tensor product $V^* \otimes V$. Contraction.
- What happens when we choose a basis.

Symmetric and alternating tensors.
- Symmetrization and anti-symmetrization. Wedge product.
- What happens when we choose a basis. Dimension of the space of alternating tensors.
- The three non-trivial alternating tensor spaces over a two-dimensional vector space.
- Duality via the $*$-operator.

Tensor bundles on a surface.
- Definition of a linear bundle over a surface $S$.
- The tangent and cotangent bundles. Higher tensor bundles. Induced metric.
- Examples: the bundle of symmetric 2-tensors. and the bundle of $k$-forms.

Covariant differentiation in a linear bundle.
- The covariant derivative of vectors extends naturally to linear bundles.
- What happens when we choose a basis.

Exterior calculus.
- The exterior differential $d$.
- The co-differential $\delta := (-1)^{e(k)} * d*$ where $k$ is the degree of the form acted upon.
- Formulas for divergence, gradient and curl.
- Relation to covariant derivatives.

Stokes Theorem.
- Chains. Boundary of a chain.
- Integration of a $k$-form over a $k$-chain.
- Stokes' formula $\int_c d\lambda = \int_{\partial c} \lambda$.

Topology and the de Rham complex.
- The de Rham complex.
- Laplacian on forms.
- Vector field decomposition.