On Being the Right Size

“The most obvious differences between different animals are differences of size, but for some reason zoologists have paid singularly little attention to them. In a large textbook of zoology before me I find no indication that the eagle is larger than the sparrow, or the hippopotamus bigger than the hare, though some grudging admissions are made in the case of the mouse and the whale. But yet it is easy to show that the hare could not be as large as a hippopotamus, or a whale as small as a herring. For every type of animal there is a most convenient size, and a large change in size inevitably carries with it a change of form”

J. B. S. Haldane
On Being the in the Right Space

“The most obvious differences between different visualizations are differences of space, but for some reason visualization scientists have paid singularly little attention to them. In a large textbook of visualization before me I find no indication that the log-log space is different than the log-linear space, or that the mercator projection is different than the azimuthal equidistant projection, though some grudging admissions are made in the case of the parallel and perspective projections. But yet it is easy to show that distances are difficult to estimate under perspective, or that data obeying a power law is easy to see in a log-log plot. For every type of visualization there is a most convenient space, and a change into the right space inevitably makes relationships clearer.”

P. Hanrahan

Topics

Cartographic projections and distortion
Graphs and lines
Phase spaces
Reorderable spaces
Cartographic Projections

Lattitude-Longitude Projection

Figure 1.3, Flattening the Earth, Snyder
Azimuthal Equidistance

Figure 3.4, Flattening the Earth, Snyder

Equi-Heading - Mercator

Figure 1.35, Flattening the Earth, Snyder
Sinusoidal Equiareal Projection

Figure 1.39a, Flattening the Earth, Snyder

Mercator Projection of Mars

Circular craters map to circles

http://astrogeology.usgs.gov/Gallery/MapsAndGlobes/mars.html#MarsMOLACoordinateMap
Figure 1.8  Airlines' view of the United States.
Maps can be scaled to suit other than distance. In this case, airline fares are used instead of miles or other linear units.
(Map courtesy of the author.)

Scale Distance by Data
From Cartography, Dent

Scale Area by Data
From Cartography, Dent
http://www.thetube.com/content/history/map.asp

Route Maps [Agrawala & Stolte]

1. Straighten wiggly lines
2. Snap turn directions to right angles
3. Expand regions with turns
4. Contract long straight roads
5. Label carefully to avoid clutter
6. Maintain overall orientation
Issues

- Choose coordinate systems that support geometric reasoning
  - Anamorphosis: Maps features to lines
- Tension between geometric properties
  - Equiarea implies not equiangular
- People tolerate distortion -- to an extent
  - Maintain important information
  - Avoid extremes
Graphs and Lines

[Diagram of graphs and lines]

Johannes Lambert – 1765 [From Tilling]
\[ \log y = \log a + x \log b \]

Power Laws e.g. Stevens Power Laws
\[ \log P = \log x + \log y \]

### From Batch to Interactive

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- **Teddy Bear** 250 GI’s
- **Kitchen Table** 10 GI’s
- **Stemware** 100 MI’s

### Anamorphosis

- Tukey and Mosteller’s pictures of power laws
- Straightening out data
- Best power law regression
Nomograms

The Rule of Three

Theory

\[
\begin{vmatrix}
  x_1(u) & y_1(u) & w_1(u) \\
  x_2(v) & y_2(v) & w_2(v) \\
  x_3(s,t) & y_3(s,t) & w_3(s,t)
\end{vmatrix} = 0
\]
From O’cagne, Le Calculi Simplifie
3D Lines Project to 2D lines

Perception of Lines
By Eye …
Linear regression ...

Linear regression w/out outlier ...
Phase Spaces

H₂O Phase Diagram
### Mitchell Cubic Filter

\[
    h(x) = \frac{1}{6} \begin{cases} 
        (12B + 9B + 6C)x^3 + (18 + 12B + 6C)x^2 + (6 + 2B)x + 1 & |x| \\
        (6B + 6C)x^3 + (12B + 30C)x^2 + (8B + 24C)x & |x| > 1 \\
    \end{cases}
\]

**Properties:**
- \( h(x) = 1 \) for \( |x| \leq 1 \)
- B-spline: (1,0)
- Catmull-Rom: (0,1/2)

From Mitchell and Netravali
Look at other figures in that paper

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### Julia and Mandelbrot Sets

\[ z^2 \leftarrow z^2 + c \]
Poincare Phase Space

Reorderable Spaces

[From Bertin]
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1. % CLIENTELE FEMALE
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3. % U.S.A.
4. % SOUTH AMERICA
5. % EUROPE
6. % M.EAST, AFRICA
7. % ASIA
8. % BUSINESSMEN
9. % TOURISTS
10. % DIRECT RESERVATIONS
11. % AGENCY
12. % AIR CREWS
13. % CLIENTS UNDER 20 YEARS
14. % 20-35
15. % 35-55
16. % MORE THAN 55
17. PRICE OF ROOMS
18. LENGTH OF STAY
19. % OCCUPANCY
20. CONVENTIONS
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Clustering Gene Expression

Nested Spaces
Barley Data and the Trellis

Yields per plot are measured
6 Sites = {Crookstein, Duluth, Grand Rapids, Morris, University Farm, Waseca}
8 Varieties = {Glabron, Manchuria, No. 457, No. 462, No. 475, Peatland, Swansota, Trebi, Velvet, Wisconsin No. 38}
2 Years = {1931, 1932}

Example from Cleveland
The Space of Polyhedra

Haresh Lalvani
Wrap-Up

Summary

On being in the right space
Spatial encoding the most important encoding
Geometric invariants of spatial transformations support geometric reasoning
“Linear” reasoning
The good and bad of distortion
Graphs and abstract spaces recent invention