CS348C – Winter 1997

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Introduction to Holography

1. Scalar diffraction theory
   – Scalar approximation
   – Wavefront propagation
   – Imaging

2. Holography
   – Recording
   – Reconstruction
   – A few types of holography

Particle (corpuscular) vs. wave theory of light

- Grimaldi (1665) described some diffraction phenomena not consistent with corpuscular theory.
- Huygens (1678) proposed wave theory of light.
- Newton (1704) is convinced light is corpuscular.
- Young (1804) does the two slit experiment.
- Fresnel (1818) uses Huygens's theory to explain diffraction and interference.
- Kirchoff (1882) and Sommerfield (1894) put the theory on sound mathematical footing.
- Today, we model light as both a particle and a wave.

Scalar Diffraction Theory

Light is comprised of propagating **E** and **H** fields which are coupled time−varying, vector fields.

If we examine any one scalar component of the **E** or **H** fields (e.g., E_X), we find that it obeys much the same laws as the other components in many cases of interest, such as propagation through free space.

Thus, an excellent aproximation is to treat light as a scalar penomenon, i.e., a time−varying *scalar* field. The theory of light based upon this approximation is called Scalar Diffraction Theory.

This theory of light applies for most conventional optical systems and yields theoretical results that agree closely with experiment

Representing the scalar field

Light consists of waves that we can describe as:

$$E_x(\mathbf{p}, t) = A_x(\mathbf{p}) \cos[2\,\pi\,\nu\,t + \varphi_x(\mathbf{p})]$$
$$E_y(\mathbf{p}, t) = A_y(\mathbf{p}) \cos[2\,\pi\,\nu\,t + \varphi_y(\mathbf{p})]$$
$$\ldots$$

Where $\mathbf{p}$ is position, A_i is amplitude, ν is the optical frequency and φ_i is relative phase.

Instead, we now use only:

$$u(\mathbf{p}, t) = A(\mathbf{p}) \cos[2\,\pi\,\nu\,t + \varphi(\mathbf{p})]$$

A more convenient notation is:

$$u(\mathbf{p}, t) = \mathrm{Re}\{\, U(\mathbf{p}) \exp[-i\,2\,\pi\,\nu\,t] \,\}$$

where:

$$U(\mathbf{p}) = A(\mathbf{p}) \exp[-i\,2\,\pi\,\varphi(\mathbf{p})]$$

Since the time variation is known a priori, we normally think exclusively in terms of the function $U(\mathbf{p})$ which we call the complex field amplitude.

For example, when adding waves:

$$\begin{aligned} u(\mathbf{p}, t) &= u(\mathbf{p}, t) + u(\mathbf{p}, t) \\ &= \mathrm{Re}\{\, U_1(\mathbf{p}) \exp[-i\,2\,\pi\,\nu\,t] \,\} + \mathrm{Re}\{\, U_2(\mathbf{p}) \exp[-i\,2\,\pi\,\nu\,t] \,\} \\ &= \mathrm{Re}\{\, U_1(\mathbf{p}) \exp[-i\,2\,\pi\,\nu\,t] + U_2(\mathbf{p}) \exp[-i\,2\,\pi\,\nu\,t] \,\} \\ &= \mathrm{Re}\{\, [U_1(\mathbf{p}) + U_2(\mathbf{p})] \exp[-i\,2\,\pi\,\nu\,t] \,\} \end{aligned}$$

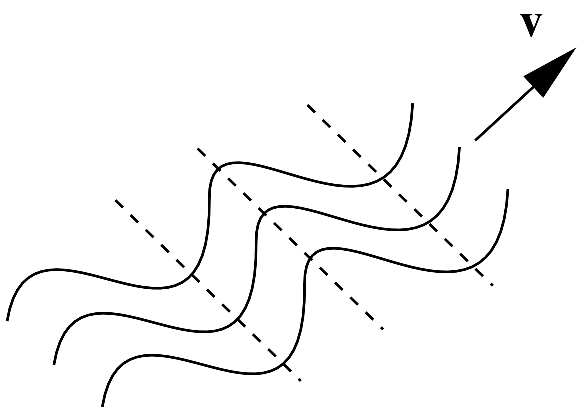
The (time average) intensity at a point in the field is:

$$I(\mathbf{p}) \sim \{A(\mathbf{p})\}^2 = UU^*$$

Plane waves and spherical waves

The field amplitude for a *uniform plane wave* is:

$U(\mathbf{p}) = A \exp[i\, k\, \mathbf{v} \bullet \mathbf{p}] = A \exp[i\, \mathbf{k} \bullet \mathbf{p}]$



where:

$k = 2\pi/\lambda$ is the wave number
$\mathbf{k} = k\, \mathbf{v}$ is the wave vector
λ is the optical wavelength
$\mathbf{v}$ is the direction of propagation

Alternatively:

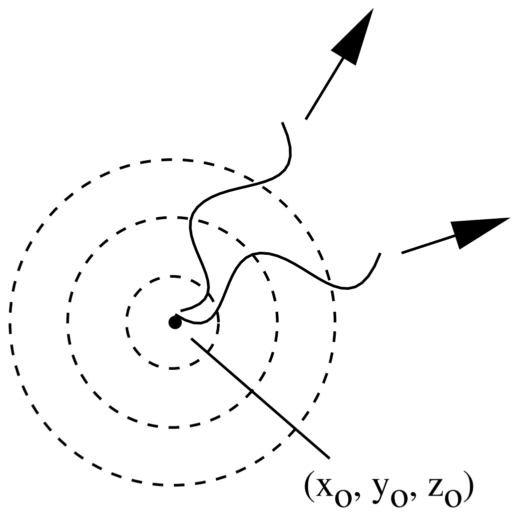
$U(\mathbf{p}) = A \exp[i\, k\, (\alpha x + \beta y + \gamma z)]$

where:

α, β, γ are the direction cosines (the components of $\mathbf{v}$)

The field amplitude of a *spherical wave* is:

$$U(\mathbf{p}) = \frac{A \exp[i\, k\, r]}{r}$$



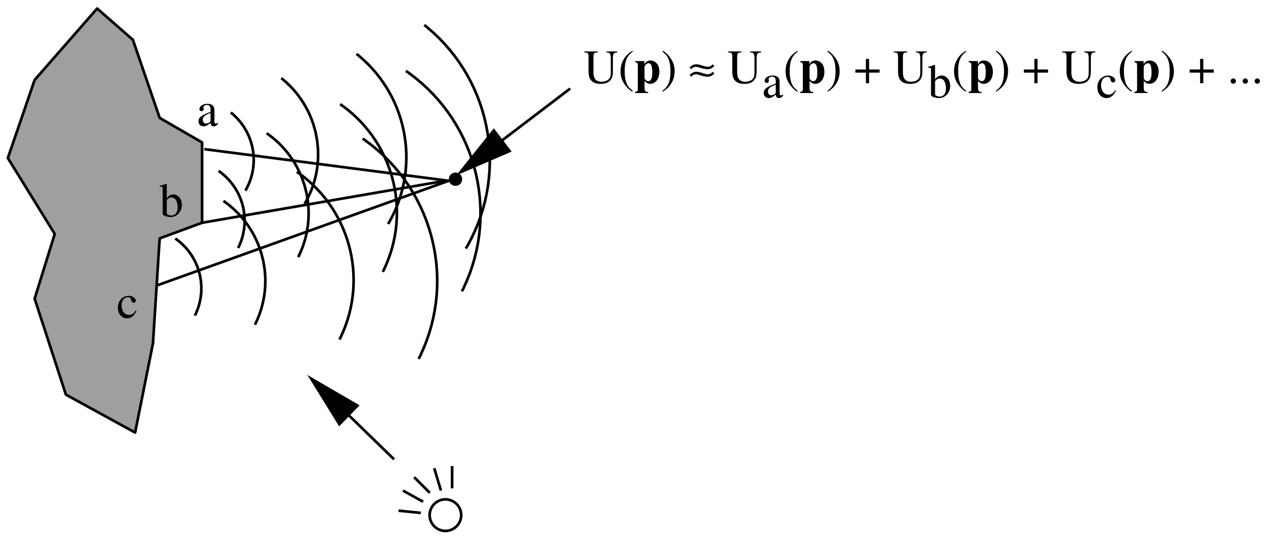
where:

$r = [(x - x_o)^2 + (y - y_o)^2 + (z - z_o)^2]^{1/2}$

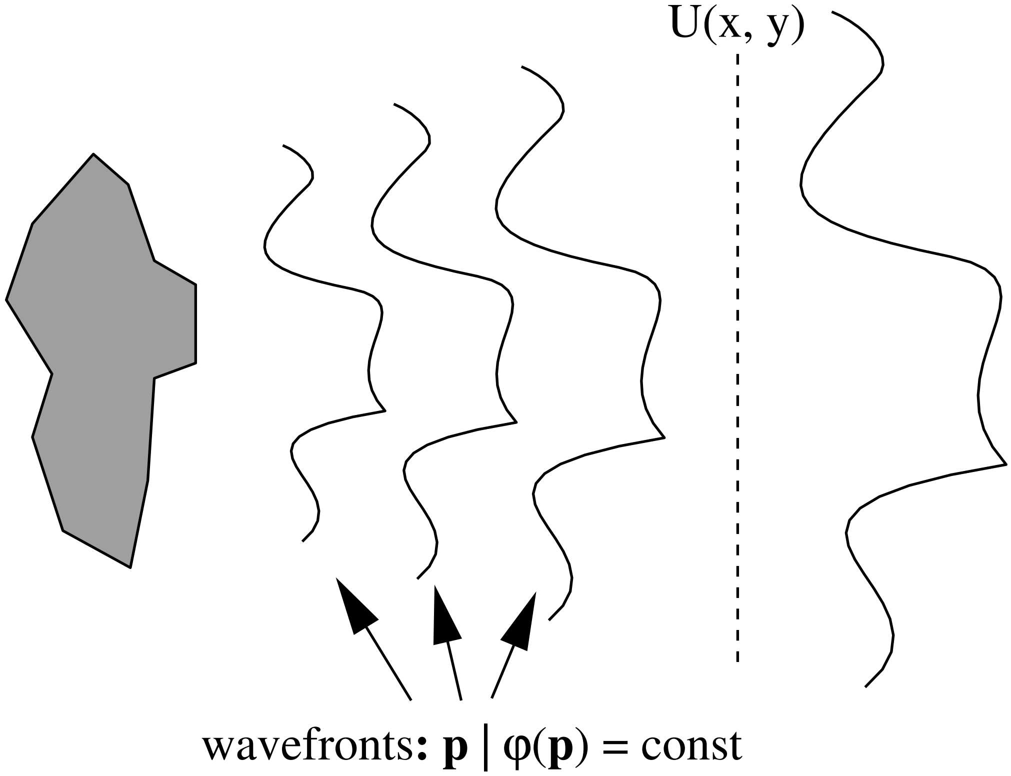
Note that the power emitted from a point source is then:

$I = UU^* = A^2/r^2$

Construction of the scalar field

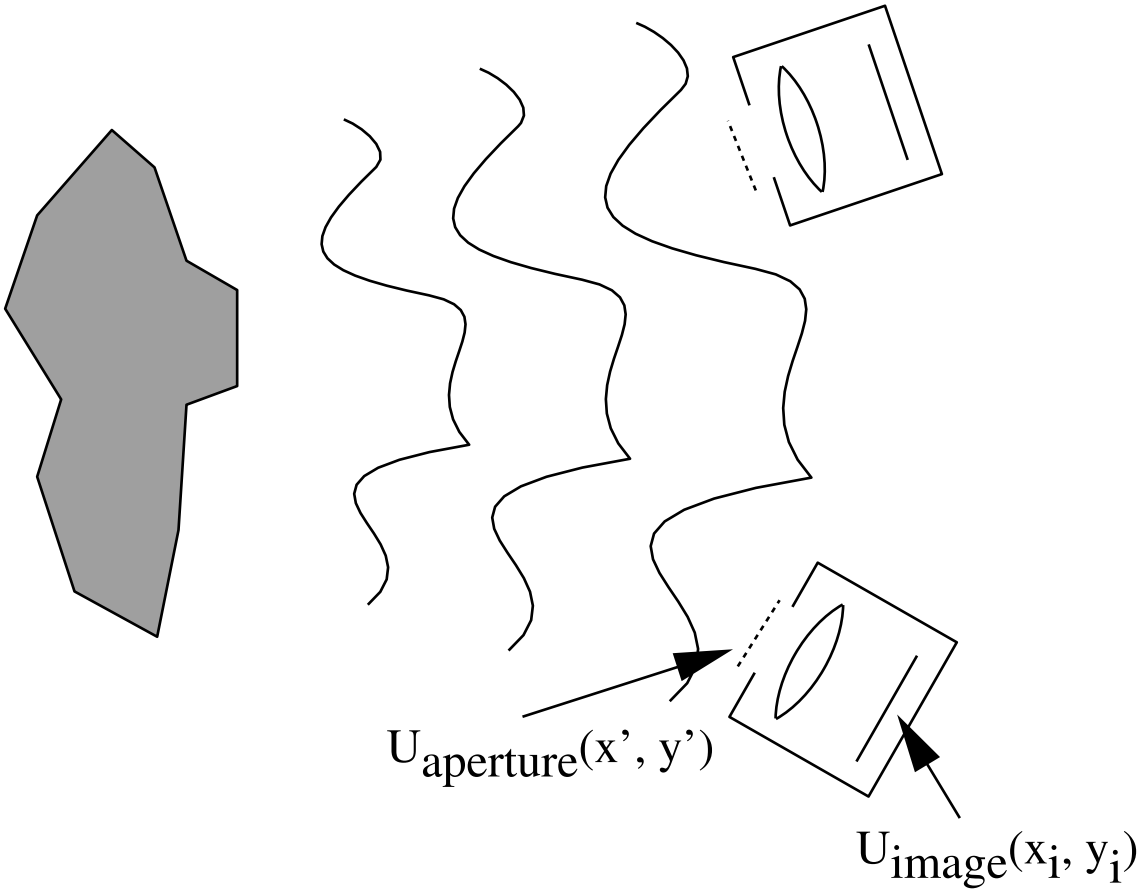


Free space propagation



Knowledge of a complete wavefront or knowledge of U(x,y) (amplitude and phase) at one plane is sufficient to reconstruct U(**p**) everywhere! ...well, almost everywhere.

Imaging the scalar field



Notice that only a small piece of the wavefront (large enough to cover the aperture) is needed to image the object.

At the image plane we record:

$$I_{image} = \left| U_{image}(x_i, y_i) \right|^2$$

Thus, the image destroys the phase and yields a picture from a single viewpoint.

Holography

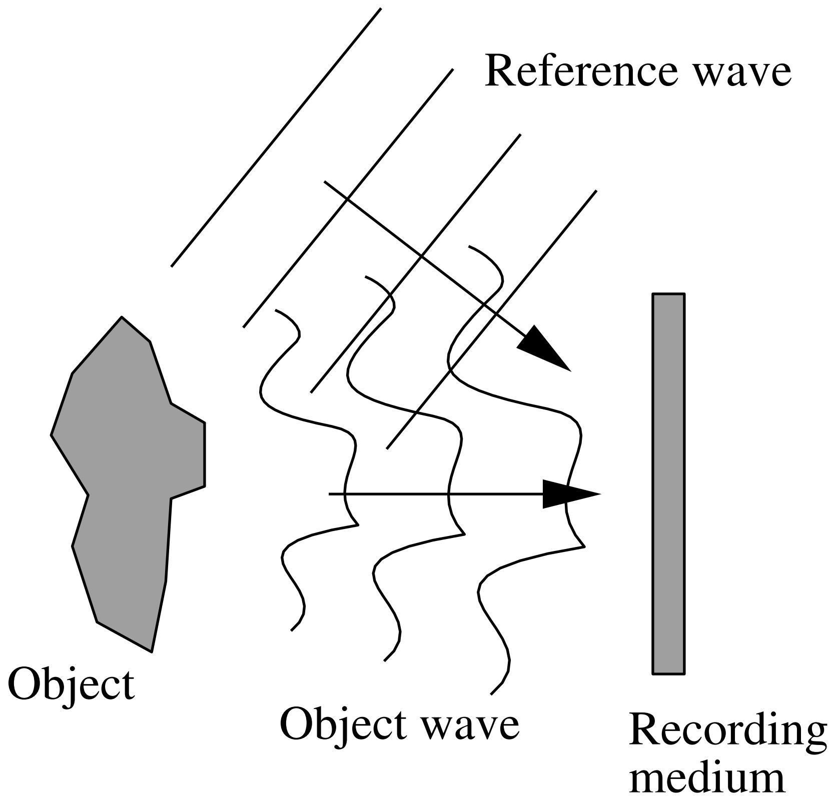
• Gabor (1948) was the first to propose a method of wavefront reconstruction, later to become known as holography. Hologram means "total recording".

• Leith and Upatnieks (1962) modified the Gabor proposal and made the first ‘‘clean’’ holograms of planar images.

• Denisyuk (1962) invented and perfected the method of thick reflection holography.

• Leith and Upatnieks (1964), with the help of newly available laser technology, create the first holograms of a 3D objects.

Recording a hologram



If we represent the complex field of the object wave at the surface of the recording medium as:

$$a(x, y) = |a(x,y)| \exp[-i\varphi(x,y)]$$

and the complex field of the reference wave as:

$$A(x,y) = |A(x,y)| \exp[-i\psi(x,y)]$$

Then, the intensity at the recording medium is:

$$\begin{aligned} I(x,y) &= |A(x,y) + a(x,y)|^2 \\ &= |A(x,y)|^2 + |a(x,y)|^2 + 2|A(x,y)||a(x,y)|\cos[\psi(x,y) - \varphi(x,y)] \end{aligned}$$

Thus, the phase of the object field has been encoded in a quantity that we can measure.

Film recording

We can rewrite the intensity at the recording medium as:

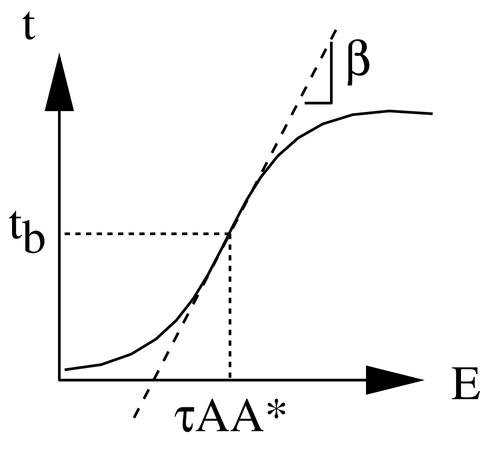
$$I = AA^* + aa^* + A^*a + Aa^*$$

For a reference that is uniform across the surface, we can regard AA^* as constant.

The film will record the intensity and yield a transparency with transmittance:

$$t(x,y) = t_b + \beta'(|a|^2 + A^*a + Aa^*)$$

where t_b is the (constant) ''bias'' transmittance due to AA^* and β' is the slope of the film's t vs. E (exposure) curve, multiplied by the exposure time, τ (i.e., $\beta' = \tau\,\beta$).



Developed film transmittance vs. exposure

If the recorded intensity has a large range, then non–linearities in film transmittance may result. This leads to aberrations in the reconstruction.

Reconstructing the hologram

Suppose we now apply a reconstruction wave, with complex amplitude B(x,y) at the plane of the developed transparency. The new wavefront will have the form:

$$U(x,y) = B(x,y)\, t(x,y)$$

$$= t_b B + \beta' aa^* B + \beta' A^* Ba + \beta' ABa^*$$

$$= U_1 + U_2 + U_3 + U_4$$

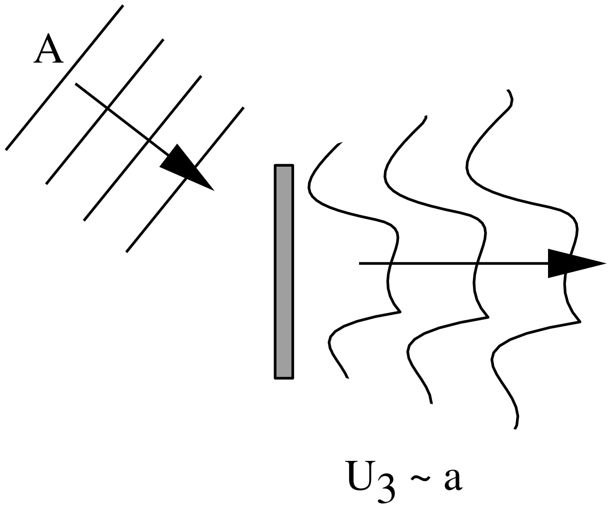
Suppose we choose B = A, then:

$$U_3 = \beta' |A|^2 a$$

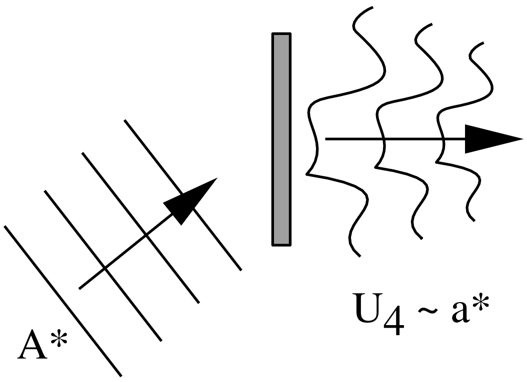
Since A is uniform and β' is constant, U_3 is a replica of the original wavefront. If we choose, B = A*, then

$$U_4 = \beta' |A|^2 a^*$$

Thus, U_4 is a replica of the *conjugate* of the original reference wave.

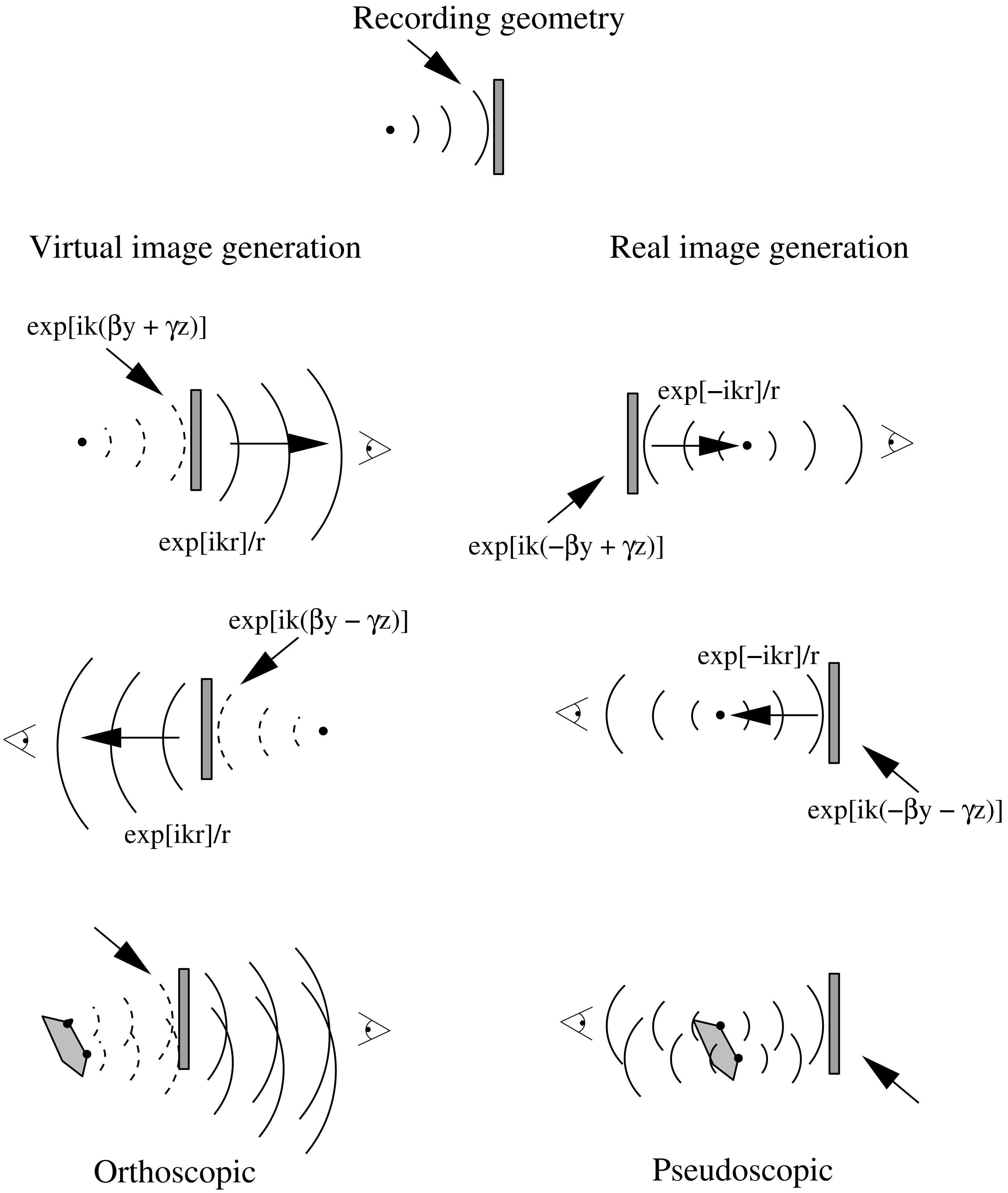


Reconstruction with reference

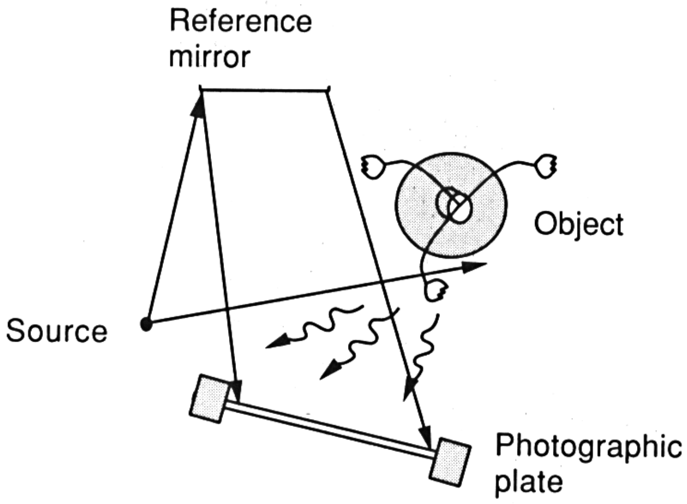


Reconstruction with conjugate reference

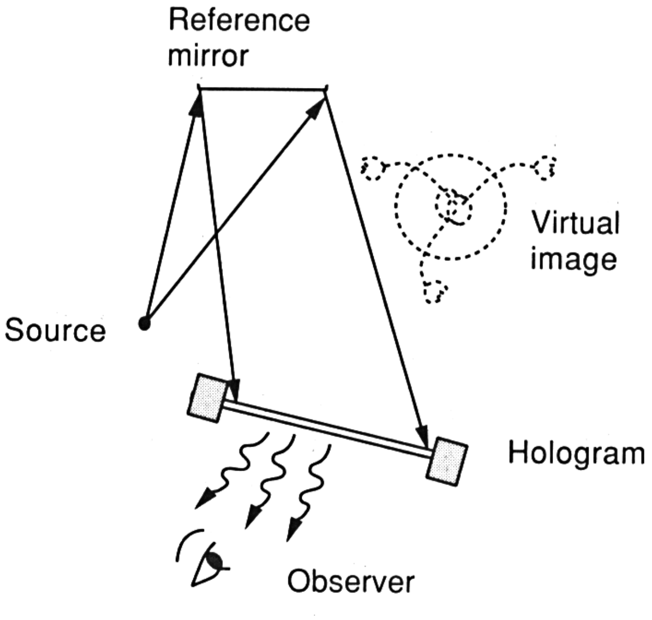
Virtual vs. real images



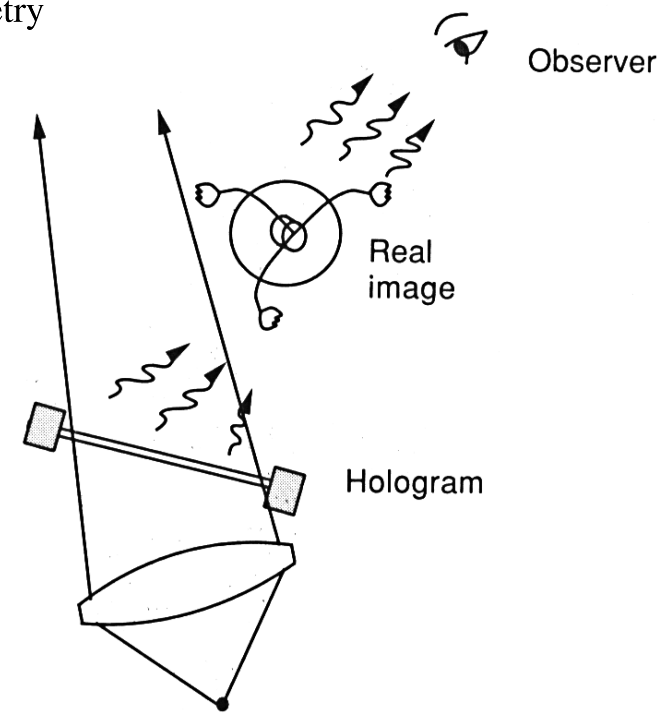
Recording and reconstruction example



Recording geometry



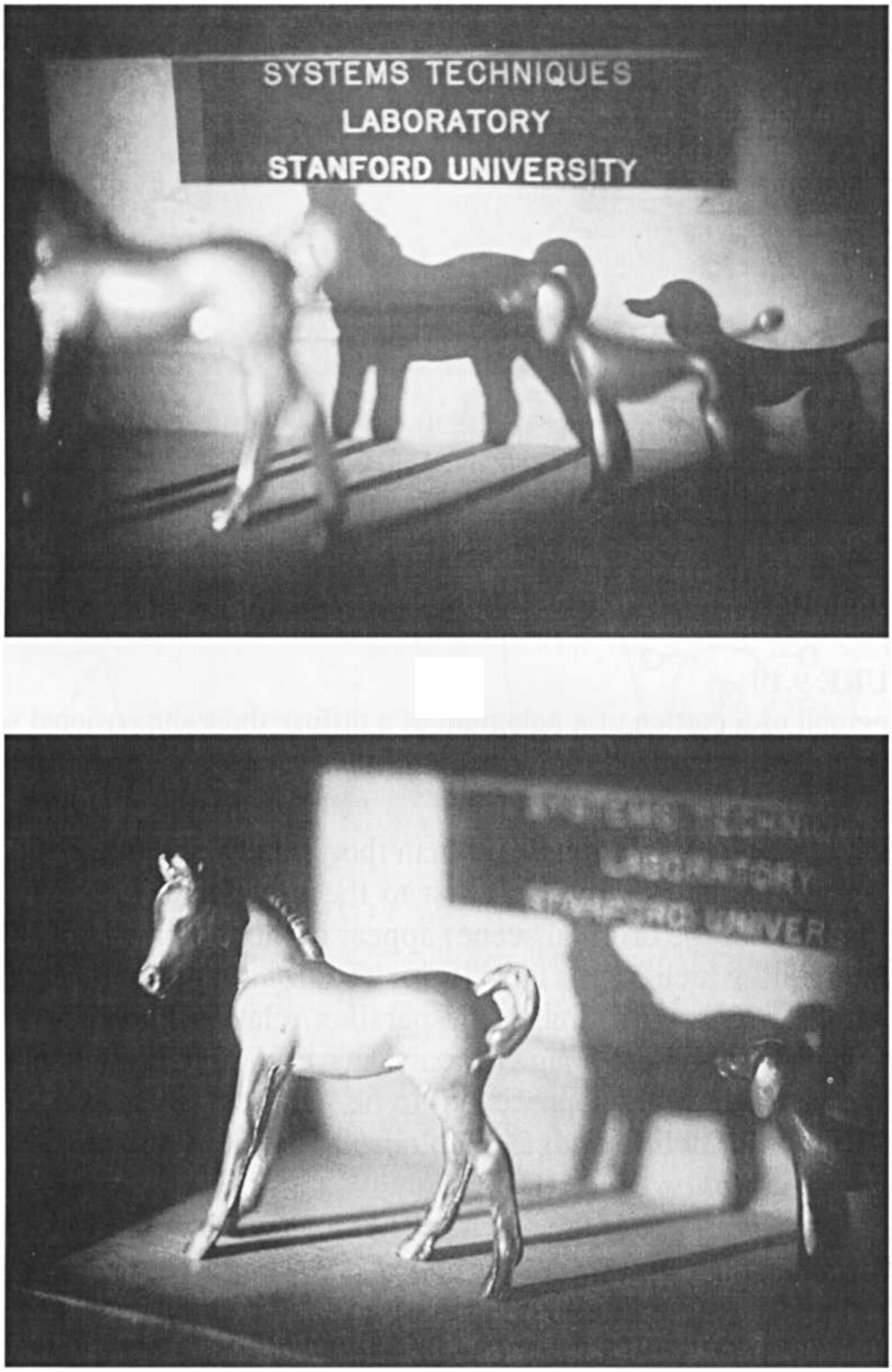
Virtual image reconstruction



Real image reconstruction

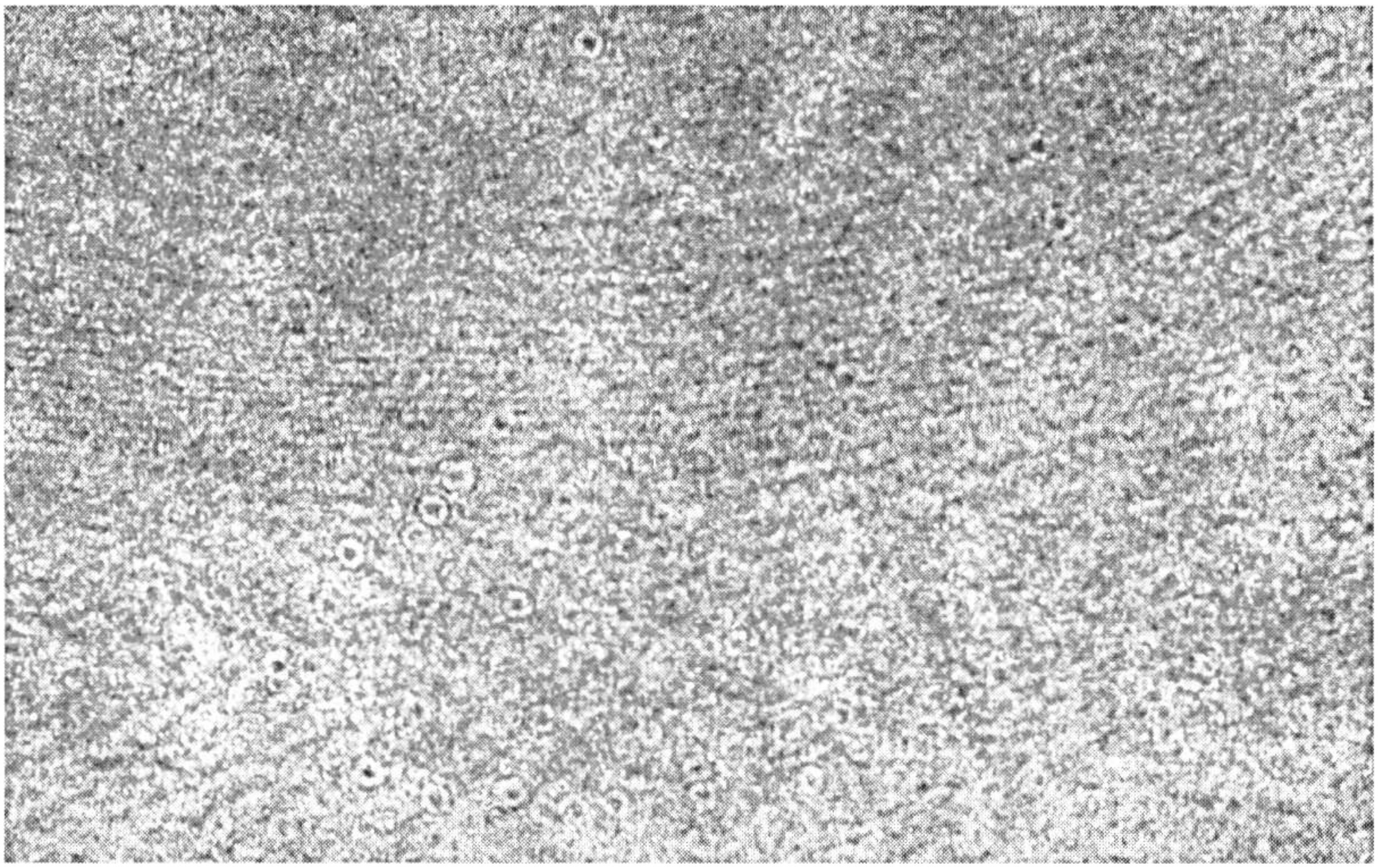
[Figure from Goodman 1996]

Two images from a transmission hologram



[Figure from Goodman 1996]

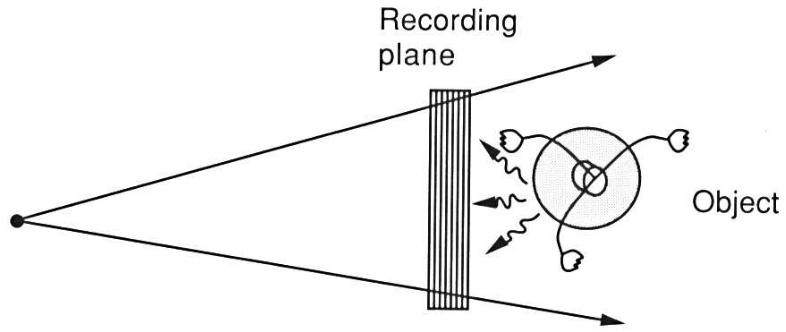
A hologram film transparency



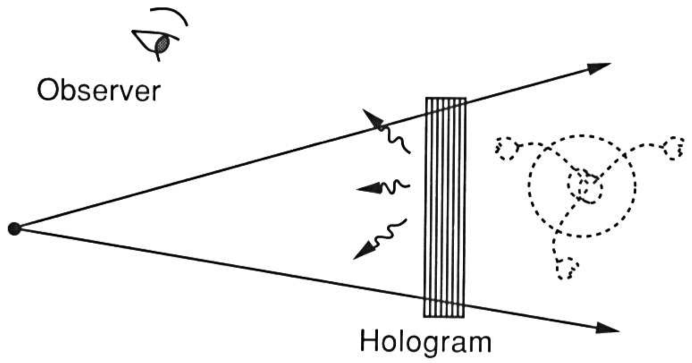
Most of the structure you see is irrelevant, arising from imperfections in the optical recording aparatus. The structure that reconstructs the images is much finer than can be seen here.

[Figure from Goodman 1996]

Reflection holograms



Recording

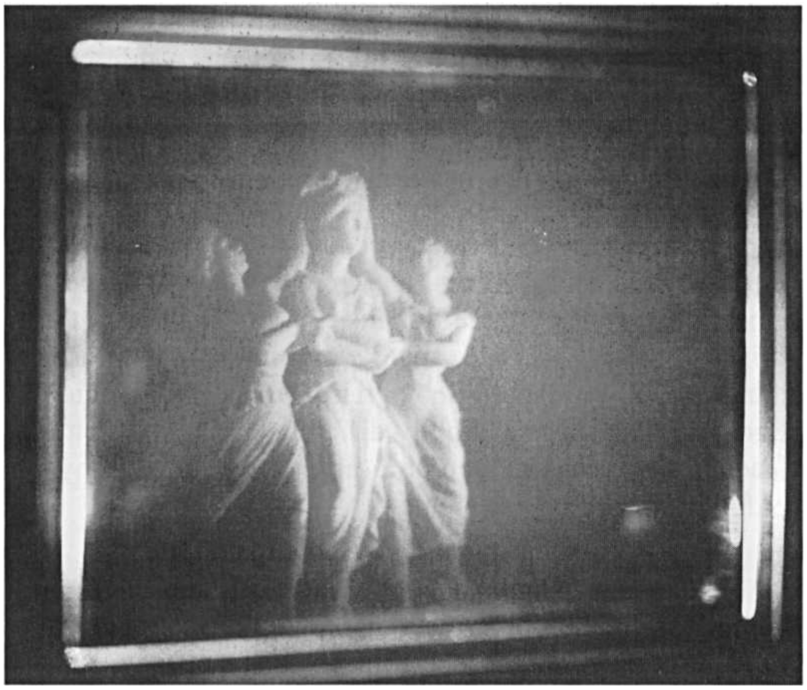


Reconstruction

Notes:

The interference pattern makes a grating *within* the volume of the thick film.

The result is viewable with white light because the volume grating acts as a wavelength filter.



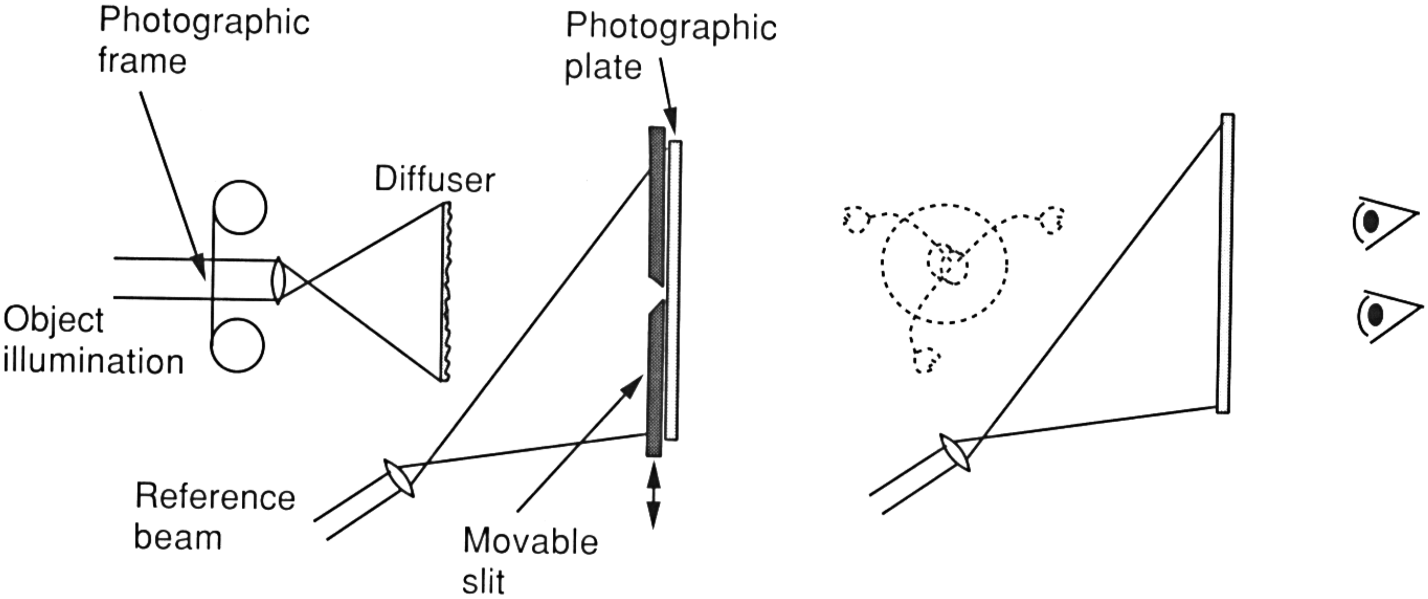
Virtual image

[Figures from Goodman 1996]

Holographic stereograms

We can record images taken from different viewpoints onto spatially separated regions of a hologram.

During reconstruction, one eye sees a reconstruction from one portion of the hologram, the other sees another image. The result is a stereo pair yielding apparent depth.



Recording   Reconstruction

Notes:

This is an ''approximation'' to a true hologram. It has limited depth of field, and quality of results depends on viewing position.

It does have the advantage that the original images can be acquired with white light, simplifying the acquisition process.

[Figure from Goodman 1996]

Bibliography

Goodman, J. *Introduction to Fourier Optics*. McGraw–Hill, San Francsico, 1996.