CS 348C — Fall 1995-6
Assignment 3 Solutions — Apostolos Lerios

Problem 1: `SoftImage 3D` Tutorial.

Here are a few conceptually distinct ways in which you used splines (curves or surfaces) while doing the indicated lessons of the `SoftImage 3D` tutorial:

- Modeled curves by direct construction: lesson 3, steps 1 through 3.
- Modeled surfaces by direct construction: lesson 1, step 2.
- Modeled surfaces by extruding splines on splines: lesson 3, step 8.
- Modeled surfaces by revolving splines about the coordinate axes: lesson 7, step 3.
- Animated the camera along a path defined by a spline curve: lesson 3, step 4.
- Deformed an object on a spline: lesson 7, step 5.
- Interpolated keyframes: lesson 7, step 8.

Problem 2: Degree-Raising Rational Curves.

1. The Bézier points of $\mathbf{q}(t)$ are

$$\mathbf{b}_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \quad \mathbf{b}_2 = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \quad \mathbf{b}_3 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix},$$

   by direct application of the given formulas. It terms of these points, we can express $\mathbf{q}(t)$ as

$$\mathbf{q}(t) = \mathbf{b}_1 B_1(t) + \mathbf{b}_2 B_2(t) + \mathbf{b}_3 B_3(t).$$

2. As we saw in class, the Bézier points of $\mathbf{c}(t)$, namely $\mathbf{a}_1, \ldots, \mathbf{a}_4$, are simply linear combinations of the Bézier points of $\mathbf{q}(t)$:

$$\begin{aligned} \mathbf{a}_1 &= \mathbf{b}_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \\ \mathbf{a}_2 &= \frac{1}{3}\mathbf{b}_1 + \frac{2}{3}\mathbf{b}_2 = \begin{pmatrix} \frac{1}{3} + \frac{2}{3}\frac{1}{\sqrt{2}} \\ \frac{1}{3} + \frac{2}{3}\frac{1}{\sqrt{2}} \\ \frac{2}{3}\frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1+\sqrt{2}}{3} \\ \frac{1+\sqrt{2}}{3} \\ \frac{\sqrt{2}}{3} \end{pmatrix} \end{aligned}$$

$$
\begin{aligned}
\mathbf{a}_3 &= \frac{1}{3}\mathbf{b}_3 + \frac{2}{3}\mathbf{b}_2 = \begin{pmatrix} \frac{1}{3} + \frac{2}{3}\frac{1}{\sqrt{2}} \\ \frac{2}{3}\frac{1}{\sqrt{2}} \\ \frac{1}{3} + \frac{2}{3}\frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1+\sqrt{2}}{3} \\ \frac{\sqrt{2}}{3} \\ \frac{1+\sqrt{2}}{3} \end{pmatrix} \\
\mathbf{a}_4 &= \mathbf{b}_3 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}.
\end{aligned}
$$

Using these points, we can express $\mathbf{c}(t)$ as

$$\mathbf{c}(t) = \mathbf{a}_1\mathcal{B}_1(t) + \mathbf{a}_2\mathcal{B}_2(t) + \mathbf{a}_3\mathcal{B}_3(t) + \mathbf{a}_4\mathcal{B}_4(t),$$

where $\mathcal{B}_1(t), \ldots, \mathcal{B}_4(t)$ are the Bernstein polynomials for cubics.

3. Projecting $\mathbf{c}(t)$ on the $w = 1$ plane produces

$$\bar{\mathbf{c}}(t) = \frac{v_1\bar{\mathbf{a}}_1\mathcal{B}_1(t) + v_2\bar{\mathbf{a}}_2\mathcal{B}_2(t) + v_3\bar{\mathbf{a}}_3\mathcal{B}_3(t) + v_4\bar{\mathbf{a}}_4\mathcal{B}_4(t)}{v_1\mathcal{B}_1(t) + v_2\mathcal{B}_2(t) + v_3\mathcal{B}_3(t) + v_4\mathcal{B}_4(t)},$$

where

$$
\begin{aligned}
\bar{\mathbf{a}}_1 &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\
\bar{\mathbf{a}}_2 &= \begin{pmatrix} \frac{1+\sqrt{2}}{3}\frac{3}{1+\sqrt{2}} \\ \frac{\sqrt{2}}{3}\frac{3}{1+\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{\sqrt{2}}{1+\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 1 \\ 0.5858 \end{pmatrix} \\
\bar{\mathbf{a}}_3 &= \begin{pmatrix} \frac{\sqrt{2}}{3}\frac{3}{1+\sqrt{2}} \\ \frac{1+\sqrt{2}}{3}\frac{3}{1+\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{2}}{1+\sqrt{2}} \\ 1 \end{pmatrix} = \begin{pmatrix} 0.5858 \\ 1 \end{pmatrix} \\
\bar{\mathbf{a}}_4 &= \begin{pmatrix} 0 \\ 1 \end{pmatrix}
\end{aligned}
$$

are the Bézier points of $\bar{\mathbf{c}}(t)$, and

$$
\begin{aligned}
v_1 &= 1 \\
v_2 &= \frac{1+\sqrt{2}}{3} = 0.8047 \\
v_3 &= \frac{1+\sqrt{2}}{3} = 0.8047 \\
v_4 &= 1
\end{aligned}
$$

are their weights.

Problem 3: Subdividing Rational Patches.

A cubic spline curve with two segments has three knots. The first and second knot always have multiplicity 3, while the multiplicity of the middle knot determines the continuity between the spline segments. In particular, for C^n

continuity, the knot is repeated $3-n$ times in the knot sequence. This means that the knot sequence has length $3+(3-n)+3=9-n$. Also, since every de Boor point of a cubic spline is labeled using three adjacent elements of the knot sequence, the spline curve has $9-n-2=7-n$ de Boor points.

A rational tensor product spline surface with two patches can be thought of as the trace of a rational curve whose Bézier points move along NURBS. If the rational curve is a cubic one, it has 4 Bézier points, and hence it takes 4 NURBS to define their paths. The shape of each of these 4 NURBS is controlled by $7-n$ de Boor points, if the two segments of the NURBS join with C^n continuity. It follows that the spline surface has $4(7-n)$ control points. In particular,

1. for C^2 continuity, the surface has 20 control points, and
2. for C^1 continuity, the surface has 24 control points, while for C^0 continuity, the surface has 28 control points.

Problem 4: Merging Rational Curves.

Every control point of a cubic spline is labeled using three adjacent elements of the knot sequence. Since the C^0 blend produces a spline with 10 de Boor points, while the other two blend types produce splines with 8 de Boor points each, the second knot sequence is the only possible one for the C^0 spline, as it contains 12 knots.

In the third sequence, knots 1 and 2 have multiplicity 2. Hence the middle segment of the spline joins both the first and the third one with C^1 continuity. It follows that this sequence best matches the spline produced by the C^1 blend. Note that we could have also used knot multiplicities to determine the best knot sequence for the C^0 blend.

The first sequence contains 6 distinct knots, only the first and last of which have non-unity multiplicities. So the associated spline consists of 5 segments, all joining with C^2 continuity. It follows that this sequence best matches the spline produced by the C^2 blend. This can be verified by merging rational patches, in which case the 5 patches of the resulting spline surface would have been easily discernible; by contrast, the C^0 and C^1 blends each produce surfaces with only 3 patches.

The following table summarizes the answer to this problem.

Blend type	Knot sequence											
C^2	0	0	0	1	1.33			1.67	2	3	3	3
C^0	0	0	0	1	1	1	2	2	2	3	3	3
C^1	0	0	0	1	1			2	2	3	3	3

Problem 5: NURBS in `SoftImage 3D`.

1. Here are a few sample commands which, for no apparent reason, the user may perform on splines but not on NURBS:

- **Draw→Zip Patches** cannot be applied on NURBS surfaces.
- **Draw→Drafting** cannot be applied on NURBS curves.
- **Surface→Four Sided** cannot be applied on NURBS curves to generate NURBS surfaces.
- **Deformation→by Spline** does not accept a NURBS curve as the deforming object.
- **Effect→Plane Clipping** cannot cut NURBS curves.
- **Draw→Attach Hole** cannot handle combinations of NURBS and splines.
- **Draw→Merge Curves** cannot be applied to a mixture of splines and NURBS curves.
- **Draw→Merge Surfaces** cannot be applied to a mixture of splines and NURBS surfaces.

2. The command is not implemented.

3. `SoftImage 3D` always sets all the weights of a NURBS circle or sphere to 1. So the resulting geometric primitive is a B-spline, which, as we saw in class, cannot *exactly* represent a circle or a sphere.

4. No, there is not. It is a poor aspect of the `SoftImage 3D` user interface.

5. No, it is not. This is most probably a typo, of course; the question was posed in jest.