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Written assignment #1 - sampling and antialiasing

Due Tuesday, April 29

(1) Prove that convolution is associative, i.e. that

$$[f(x) * g(x)] * h(x) = f(x) * [g(x) * h(x)].$$

(2) The triangle function of width 2 and unit area is frequently used as a filter in computer graphics. It is defined as:

$$\Lambda(x) = \begin{cases} 1 + x & -1 \le x < 0 \\ 1 - x & 0 \le x < 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Using the definition of convolution, show that the convolution of two rect functions yields the triangle function, i.e. that $\Pi(x) * \Pi(x) = \Lambda(x)$.

(b) Prove that reconstructing a sampled 1D signal by convolving it with the triangle function $\Lambda(x)$ corresponds to linearly interpolating its samples. Assume that the samples are taken one unit apart.

(3) Intensity I on a CRT is typically related to frame buffer pixel value V by the power function $I = V^{\gamma}$ for some value of gamma. If we antialias a polygon edge using one of the methods described in class and display it on a CRT without correcting for this power function, typically by loading an inverse power function into the hardware colormap, then the image will be visually unsatisfactory.

(a) What will the artifacts look like? Sketch them.

(b) The presence of an uncorrected power function violates one of the premises underlying the sampling and reconstruction pipeline presented in class. Which one?

(4) Consider a 1-D sinusoidal signal with a period of 2.2 pixels regularly sampled once per pixel and stored in a frame buffer.

(a) Sketch a graph of the sampled signal.

(b) Sketch a graph of the signal as it would appear if displayed on a standard CRT.

(c) We appear to be sampling above the Nyquist limit, yet the displayed image contains visible artifacts. Explain.

(5) Consider an axis-aligned rectangle moving horizontally at a constant velocity. Suppose that we implement analytic spatiotemporal filtering by convolving the rectangle with a 3D rect function $\Pi(x, y, t)$ followed by sampling. This process reduces to calculating a volume per pixel per frame. Make a carefully labeled sketch depicting this situation and showing the volume to be computed.

(Turn over for extra credit problems!)

(6) **For extra credit:** In problem 4, working from the available samples, and ignoring the physical limitations of CRTs, is there *any* way to eliminate these artifacts in the displayed image?

(7) **For more extra credit:** In problem 5, consider multiple rectangles having a fixed occlusion ordering and moving horizontally at different velocities. To render this situation correctly, pixels will occasionally receive contributions from more than one rectangle. How does this complicate your solution to part (a)? Sketch it.