To The Rendering Equation

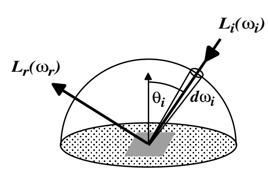
Questions

1. How is light measured?
2. How is the spatial distribution of light energy described?
3. How is reflection from a surface characterized?
4. What are the conditions for equilibrium flow of light in an environment?

Illumination Models

To evaluate the reflection equation the illumination must be specified or computed.

$$L_r(x,\omega_r) = \int_{H^2} f_r(x,\omega_i \to \omega_r)\, L_i(x,\omega_i) \cos\theta_i \, d\omega_i$$



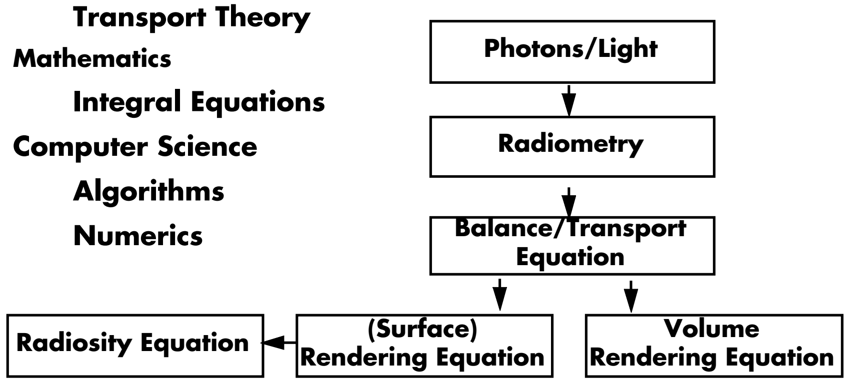
- Direct (*local*) illumination
  - Primary light sources
  - No shadows, no occlusion
  - No integration (point sources)
- Indirect (*global*) illumination
  - Shadows and occlusion
  - Secondary light sources, interreflections
  - Area or directional integration
  - Complete accounting of light energy

The Grand Scheme

Physics
- **Transport Theory**

Mathematics
- **Integral Equations**

Computer Science
- **Algorithms**
- **Numerics**



Balance Equation

Accountability

[*outgoing*] - [*incoming*] = [*emitted*] - [*absorbed*]

Macro to micro phase space

$$\Phi_o - \Phi_i = \Phi_e - \Phi_a \quad [W]$$

$$B(x) - E(x) = B_e(x) - B_a(x) \quad \left[\frac{W}{m^2}\right]$$

$$L_o(x,\omega) - L_i(x,\omega) = L_e(x,\omega) - L_a(x,\omega) \quad \left[\frac{W}{m^2 sr}\right]$$

Energy (Power) Balance

- **Micro, or local level**

  The energy flowing into a small region of phase space must equal the energy flowing out.

- **Macro, or global level**

  The total light energy put into the system must equal the energy leaving the system (usually, via heat).

Surface Balance Equation

[*outgoing*] = [*emitted*] + [*reflected*] + [*transmitted*]

$$L_o(x,\omega) = L_e(x,\omega) + L_r(x,\omega) + L_t(x,\omega)$$

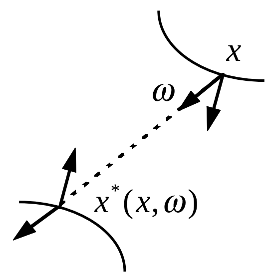
1. **Ignore transmission**
2. **Reflection equation relates incident to reflected**

$$L_o(x,\omega_o) = L_e(x,\omega_o) + \int_{H^2} f_r(x,\omega_i \to \omega_o)\, L_i(x,\omega_i) \cos\theta_i \, d\omega_i$$

Couple Balance Equations

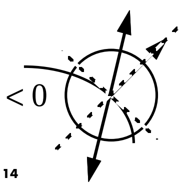
1. Relationship between incoming and outgoing radiance

$$L_i(x^*(x,\omega),-\omega) = L_o(x,\omega)$$



$$L_i(x,-\omega) = L(x,\omega) \quad \omega \bullet n < 0$$
$$L_o(x,\omega) = L(x,\omega) \quad \omega \bullet n > 0$$

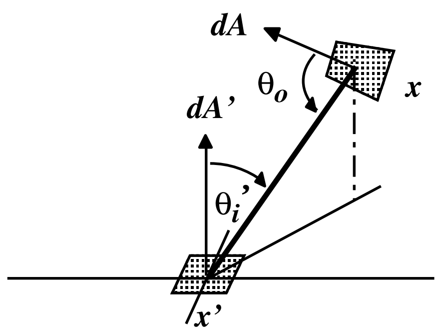
$H_-^2 \quad \omega \bullet n < 0$



$H_+^2 \quad \omega \bullet n > 0$

Two Point Geometry

2. Relationship between solid angle and source surface



$$d\omega' = \frac{\cos\theta_o}{\|x - x'\|^2} dA$$

$$\cos\theta_i' \, d\omega' \, dA' = \frac{\cos\theta_i' \cos\theta_o}{\|x - x'\|^2} dA \, dA'$$

The Rendering Equation

1. Directional integral [Immel, Cohen, Greenberg 1986]

$$L(x,\omega_o) = L_e(x,\omega_o) + \int_{H_+^2} f_r(x,\omega_i \to \omega_o)\; L(x^*(x,\omega_i),\omega_i)\, \cos\theta_i \, d\omega_i$$

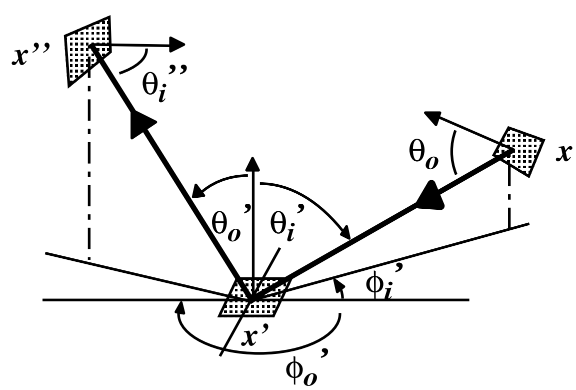
2. Surface integral

$$L(x,\omega_o) = L_e(x,\omega_o) + \int_{M^2} f_r(x,\omega_i(x'-x) \to \omega_o)\, G(x,x') L(x',\omega_o'(x-x'))\; dA'$$

$$G(x,x') = \frac{\cos\theta_o' \cos\theta_i}{\|x-x'\|^2} V(x,x')$$

Visibility term $V(x,x')$

Three Point Transport [Kajiya 1986]



$$L(x',x'') = L_e(x',x'') + \int_{M^2} f_r(x \to x' \to x'')\; G(x,x')\, L(x,x')\; dA$$

The Radiosity Equation

Assume <u>only</u> diffuse reflection

1. $f_r(x, \omega_i \to \omega_o) = f_{r,d}(x)$

2. $L(x, \omega) = B(x) / \pi$

Two Point Transport

$$B(x) = B_e(x) + f_{r,d}(x) \int B(x')\, G(x, x')\, dA'$$

$$B(x) = E(x) + \rho(x) \int B(x')\, F(x, x')\, dA'$$

<u>Form factor</u>: The percentage of light leaving *dx′* that arrives at *dx*

$$F(x, x') = G(x, x') / \pi$$

Integral Equations and Operators

Fredholm integral equation of the 2nd kind

$$f(x) = g(x) + \int k(x, y)\, f(y)\, dy$$

Integral operator

$$(\mathrm{K}f)(x) = \int k(x, y)\, f(y)\, dy$$

$$(\mathrm{I} - \mathrm{K})\, f = g$$

Formal Solution

$$(\mathrm{I}-\mathrm{K})f = g \quad \Rightarrow \quad f = (\mathrm{I}-\mathrm{K})^{-1} g$$

Invert

$$\frac{1}{1-x} = 1 + x + x^2 + \ldots \text{for} |x| < 1$$

$$\begin{aligned}(\mathrm{I}-\mathrm{K})^{-1} &= \mathrm{I} + \mathrm{K} + \mathrm{K}^2 + \ldots \\ &= \mathrm{I} + \mathrm{K}(\mathrm{I} + \mathrm{K}(\mathrm{I} + \ldots\end{aligned}$$

Verify:

Physical interpretation