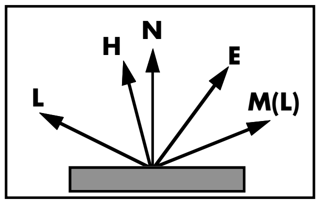
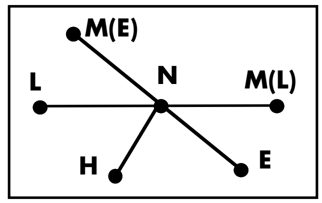
Reflection Geometry





$$\cos\theta_i = \mathbf{L} \bullet \mathbf{N} \qquad \cos\theta_g = \mathbf{E} \bullet \mathbf{N}$$

$$\cos\theta_r = \mathbf{E} \bullet \mathbf{N} \qquad \cos\theta_s = \mathbf{R}(\mathbf{L}) \bullet \mathbf{E} = \mathbf{L} \bullet \mathbf{R}(\mathbf{E})$$

$$\cos\theta_{s'} = \mathbf{H} \bullet \mathbf{N}$$

Microfacet Distributions

$$d^2 A(\omega) = A(\omega)\, d\omega$$

$$dS = \int_{2\pi} A(\omega)\, d\omega$$

$$dA = \int_{2\pi} A(\omega) \cos\theta\, d\omega$$

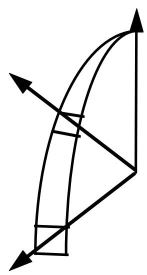
$$D(\omega) = A(\omega) / dA$$

$$\int_{2\pi} D(\omega) \cos\theta\, d\omega = 1$$

Solid Angle Distributions

$$d\omega_r = \sin\theta_r \, d\theta_r \, d\varphi_r$$

$$d\omega_m = \sin\theta_m \, d\theta_m \, d\varphi_m$$

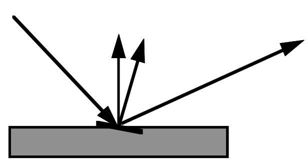


$$\theta_r = 2\theta_m$$

$$\varphi_r = \varphi_m$$

$$d\omega_r = \sin\theta_r \, d\theta_r \, d\varphi_r = (2\cos\theta_m \sin\theta_m)(2d\theta_m)(d\varphi_r)$$
$$= 4\cos\theta_m \, d\omega_m$$

Microfacet BRDF



$$dA' = D(\omega')\, d\omega' \, dA$$

$$f_r \equiv \frac{dL_r}{dE_i} = \frac{D(\omega')}{4\cos\theta_i \cos\theta_r}$$

$$d^2\Phi_m = L_i(\omega_i)\cos\theta' \, d\omega_i \, dA'$$

$$d^2\Phi_r = L_r(\omega_r)\cos\theta_r \, d\omega_r \, dA$$

$$d^2\Phi_r = d^2\Phi_m$$

$$\therefore L_r(\omega_r) = \frac{L_i(\omega_i)\cos\theta' \, d\omega_i \, dA'}{\cos\theta_r \, d\omega_r \, dA}$$
$$= \frac{L_i(\omega_i)\cos\theta' \, d\omega_i \, D(\omega')\, d\omega'}{\cos\theta_r \, d\omega_r}$$
$$= \frac{L_i(\omega_i) D(\omega')\, d\omega_i}{4\cos\theta_r}$$

Microfacet Distribution Functions

Blinn

$$D_1(\alpha) = \cos^{c_1} \alpha \qquad c_1 = \frac{\ln 2}{\ln \cos \beta}$$

Torrance-Sparrow

$$D_2(\alpha) = e^{-(c_2 \alpha)^2} \qquad c_2 = \frac{\sqrt{2}}{\beta}$$

Trowbridge-Reitz

$$D_3(\alpha) = \left(\frac{c_3^2}{\cos^2 \alpha (c_3^2 - 1) - 1} \right) \qquad c_3 = \left(\frac{\cos^2 \beta - 1}{\cos^2 \beta - \sqrt{2}} \right)^2$$

Beckmann

$$D_4(\alpha) = \frac{1}{\pi^{1/2} m^2 \cos^2 \alpha} e^{-\frac{\tan^2 \alpha}{m^2}}$$

Gaussian Rough Surface

Beckmann

$$p(z) = \frac{1}{(2\pi)^{1/2} \sigma} e^{-\frac{z^2}{2\sigma^2}}$$

$$D(\alpha) = \frac{1}{\pi^{1/2} m^2 \cos^2 \alpha} e^{-\frac{\tan^2 \alpha}{m^2}}$$

Smith

$$S(\theta_i, \theta_r) = S(\theta_i) S(\theta_r)$$

$$S(\theta) = \frac{1 - \frac{1}{2} \mathrm{erfc}(\tan\theta / \sqrt{2} m)}{1 + \frac{1}{2} ((\frac{2}{\pi})^{1/2} \frac{\mathrm{m}}{\tan\theta} \mathrm{e}^{-\tan^{2\theta} / 2m^2} - \mathrm{erfc}(\tan\theta / \sqrt{2} m))}$$

__Theory self-consistent:__
The sum of the areas of the illuminated surface projected onto the plane normal to the direction of incidence is independent of the roughness of the surface, and equal to the projected area of the underlying mean place.

Schlick approximation

$$S'(\theta) = \frac{\cos\theta}{\cos\theta r + r(1 - \cos\theta)}$$

Self-Shadowing: V-Groove Model

Assumptions (Torrance-Sparrow)

1. **Symmetric, longitudinal, isotropically-distributed**
2. **Upper edges lie in plane**

Masking of reflection, Shadowing of incident

$$G_a = 1$$

$$G_b = 2\mathbf{N} \bullet \mathbf{H} \mathbf{N} \bullet \mathbf{E} / \mathbf{E} \bullet \mathbf{H}$$

$$G_c = 2\mathbf{N} \bullet \mathbf{H} \mathbf{N} \bullet \mathbf{L} / \mathbf{L} \bullet \mathbf{H}$$

$$G = \min(G_a, G_b, G_c)$$