Questions

1. How is light measured?
2. How is the spatial distribution of light energy described?
3. $\boxed{\text{How is reflection from a surface characterized?}}$
4. What are the conditions for equilibrium flow of light in an environment?

Reflection Models

Definition: Reflection is the process by which light incident on a surface interacts with the surface such that it leaves on the incident side without change in frequency.

Properties

- Directional distribution
- Spectra and Color
- Polarization

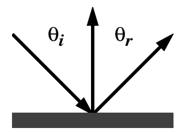
Theories

- Phenomenological
- Physical

Types of Reflection Functions

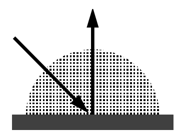
Ideal Specular

- **Reflection Law**
- **Mirror**



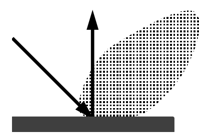
Ideal Diffuse

- **Lambert's Law**
- **Matte**



Specular

- **Glossy**
- **Directional diffuse**



Computer Graphics Models

For example, SGI shading model

$$L_r = k_a L_a + \sum \left(k_d (\mathbf{N} \bullet \mathbf{L}_i) + k_s (\mathbf{N} \bullet \mathbf{H}_i)^s \right) L_i$$

Terms:

- **Ambient - Hack!**
- **Diffuse**
  Bouknight, Gouraud
- **Specular**
  Phong, Blinn

Processes

Aspects

- **Surface vs. subsurface interaction**
- **Single vs. multiple interactions**
- **Homogeneous vs. heterogenous materials**

Examples

- **Varnish**
- **Paint**
- **Metal**
- **Cloth**

Later we will consider a complete accounting including:

reflected, transmitted, emitted and aborbed light

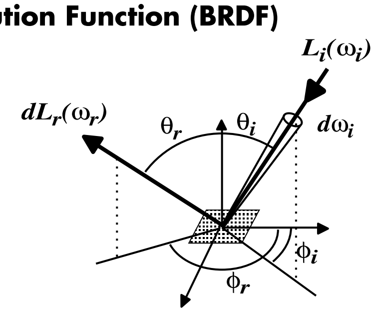
The BRDF

Bidirectional Reflectance-Distribution Function (BRDF)

$$d\Phi_i = L_i(\omega_i)\cos\theta_i d\omega_i \, dA$$

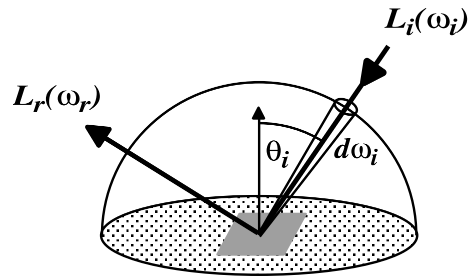
$$d^2\Phi_r = dL_r(\omega_i,\omega_r)\cos\theta_r d\omega_r \, dA$$

$$d\rho = \frac{d^2\Phi_r}{d\Phi_i} = \frac{dL_r(\omega_i,\omega_r)\cos\theta_r \, d\omega_r}{L_i(\omega_i)\cos\theta_i d\omega_i}$$



$$f_r(x,\omega_i \to \omega_r) \equiv \frac{d\rho}{\cos\theta_r \, d\omega_r} = \frac{dL_r(\omega_i,\omega_r)}{dE(\omega_i)} = \frac{dL_r(\omega_i,\omega_r)}{L_i(\omega_i)\cos\theta_i \, d\omega_i} \quad \left[\frac{1}{sr}\right]$$

The Reflection Equation



$$L_r(x,\omega_r) = \int_{2\pi} f_r(x,\omega_i \to \omega_r)\, L_i(x,\omega_i) \cos\theta_i \, d\omega_i$$

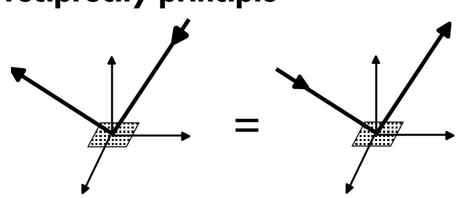
Linearity:

1. Doubling the incident light, doubles the reflected light.

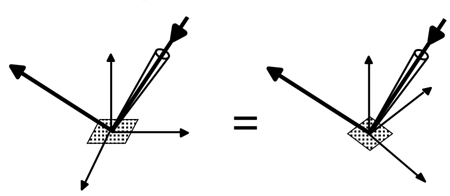
2. Several sources behave like the sum of each source.

Properties of BRDF's

1. Helmholtz reciprocity principle



2. Isotropic vs. anisotropic



Generalizations

Bidirectional Scattering-Surface Reflectance-Distribution Function (BSSRDF)

- ***t***, λ– **Flourescence and phosphorescence**

$$f_r(x_i \to x_r, \omega_i \to \omega_r, t_i \to t_r, \lambda_i \to \lambda_r)$$

Bidirectional transmittance-distribution function (BTDF)

$$f_t(x, \omega_i \to \omega_r, t, \lambda)$$

Simplifications

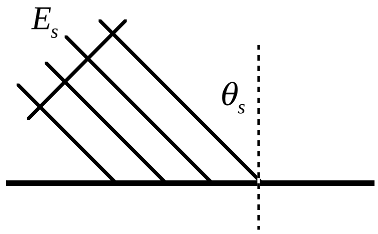
1. Eliminate area integral

- **Ambient light**
- **Point source**
- **Distant source**

2. Ideal materials

- **Perfect reflection and refraction**
- **Perfect diffuse**

Distant Source



$$E = \int L \cos\theta \, d\omega = E_s \cos\theta_s$$

$$L(\theta,\varphi) = E_s \delta(\cos\theta - \cos\theta_s)\delta(\varphi - \varphi_s)$$

$$\boxed{\begin{array}{l} \int \delta(x)\,dx = 1 \\ \int f(x)\delta(x-a)\,dx = f(a) \end{array}}$$

$$\begin{aligned} &\int L(\theta,\varphi)\cos\theta \, d\cos\theta \, d\varphi \\ &= \int E_s \delta(\cos\theta - \cos\theta_s)\delta(\varphi - \varphi_s)\cos\theta \, d\cos\theta \, d\varphi \\ &= E_s \cos\theta_s \end{aligned}$$

The Reflectance

<u>Definition</u>: A *reflectance* is a ratio of reflected to incident flux

$$\rho(\omega_i \to \omega_r) \equiv \frac{d\Phi_r}{d\Phi_i} = \frac{\int\limits_{\Omega_r} L_r(\omega_r)\cos\theta_r \, d\omega_r \, dA}{\int\limits_{\Omega_i} L_i(\omega_i)\cos\theta_i \, d\omega_i \, dA} \text{ [dimensionless]}$$

$$= \frac{\int\limits_{\Omega_r}\int\limits_{\Omega_i} f_r(\omega_i \to \omega_r)\cos\theta_i \, d\omega_i \cos\theta_r \, d\omega_r}{\int\limits_{\Omega_i} \cos\theta_i \, d\omega_i}$$

- **Derivation assumes uniform incident radiance**
- **All experiments measure reflectances**
- **Conservation of energy: *0* $\leq \rho \leq$ *1* vs. *0* $\leq f_r \leq \infty$**
- **Units: ρ [dimensionless], f_r [1/steradians]**

Directional-Hemispherical Reflectance

Definition: $\rho(\omega_i \to 2\pi) = \int_{2\pi} f_r(\omega_i \to \omega_r) \cos\theta_r \, d\omega_r$

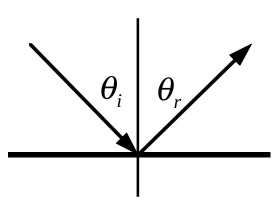
1. Reciprocity $\rho(\omega_i \to 2\pi) = \rho(2\pi \to \omega_r)$

2. Isotropy $\rho(\omega_i \to 2\pi) = \rho(2\pi \to \omega_r) = \rho(\theta)$

3. Normal reflectance $\rho_n = \rho(0)$

Normalization condition required to conserve energy

Ideal Specular Reflection



$$f_{r,s}(\theta_r,\varphi_r;\theta_r,\varphi_r) = \delta(\cos\theta_r - \cos\theta_i)\delta(\varphi_r - \varphi_i \pm \pi)$$
$$= \frac{\delta(\theta_r - \theta_i)\delta(\varphi_r - \varphi_i \pm \pi)}{\sin\theta_i}$$
$$= 2\delta(\sin^2\theta_r - \sin^2\theta_i)\delta(\varphi_r - \varphi_i \pm \pi)$$

$$L_{r,s}(\theta_r,\varphi_r) = L_i(\theta_r,\varphi_r \pm \pi)$$
$$\theta_i = \theta_r$$
$$\varphi_i = \varphi_r \pm \pi$$

$$\boxed{\delta(f(x) - f(a)) = \frac{\delta(x-a)}{f'(a)}}$$

Ideal Diffuse Reflection

Assume light is equally likely to be reflected in any output direction (independent of input direction).

$$
\begin{aligned}
L_r(\omega_r) &= \int_{2\pi} f_{r,d}\, L_i(\omega_i)\cos\theta_i\, d\omega_i \\
&= f_{r,d} \int_{2\pi} L_i(\omega_i)\cos\theta_i\, d\omega_i \\
&= f_{r,d}\, E
\end{aligned}
$$

Lambert's Cosine Law

1. **The reflected radiance L is constant and equal in all reflected directions.**
2. **Therefore $L = B/\pi$.**
3. **The magnitude of the reflected radiance L equals $f_{r,d}E$.**
4. **All reflectances are equal: $\rho_d = \pi f_{r,d}$**
5. **The total reflectance equals the radiosity over the irradiance B/E.**

“Diffuse” Reflection

Theoretical

- **Bouguer - Special micro-facet distribution**
- **Seeliger - Subsurface reflection**
- **Multiple surface or subsurface reflections**

Experimental

- **Pressed magnesium oxide powder**
- **Almost never valid at high angles of incidence**

Paint manufactures attempt to create ideal diffuse