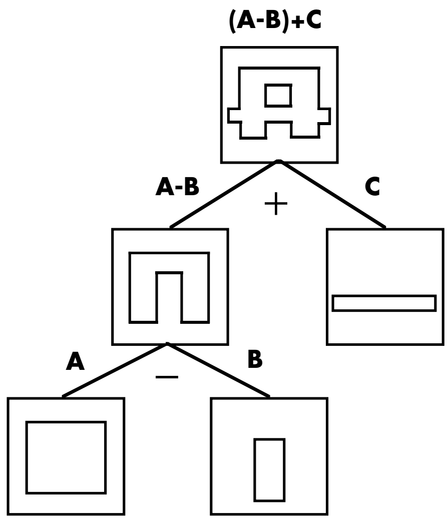
Constructive Solid Geometry

Primitives

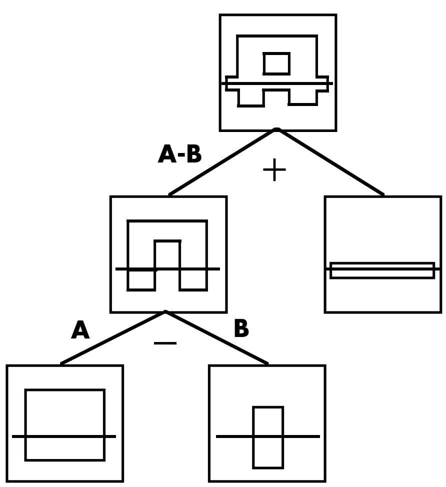
- Point Classification

Spatial set operations

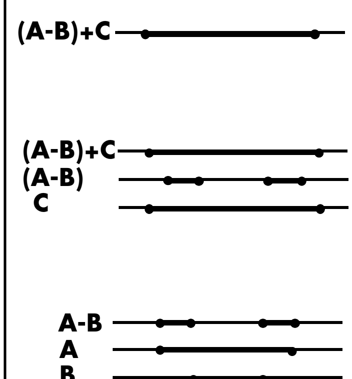
- Intersection
- Union
- Difference



Roth Algorithm



Reduce to 1D calculation
1. Sort
2. Merge

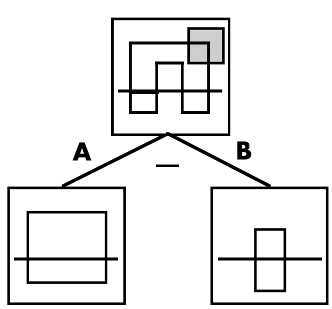


Point Classification

1. Classify ray origin
2. Proceed along the ray
   Incrementally update classification

CSG Optimizations

1. Tree Pruning (Broonsvoort et al., Woodwork)
   - Within active region simplify CSG tree



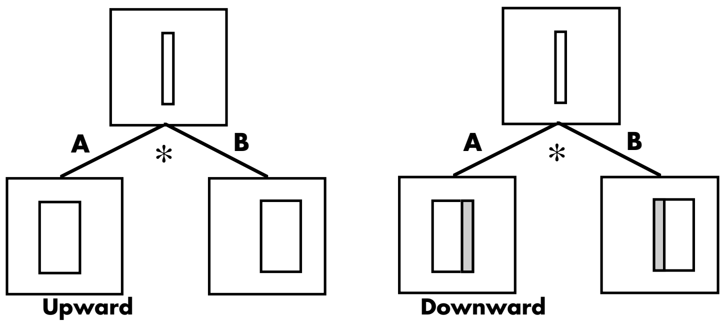
A*B
- B=0 0
- B=1 A

A+B
- B=0 A
- B=1 1

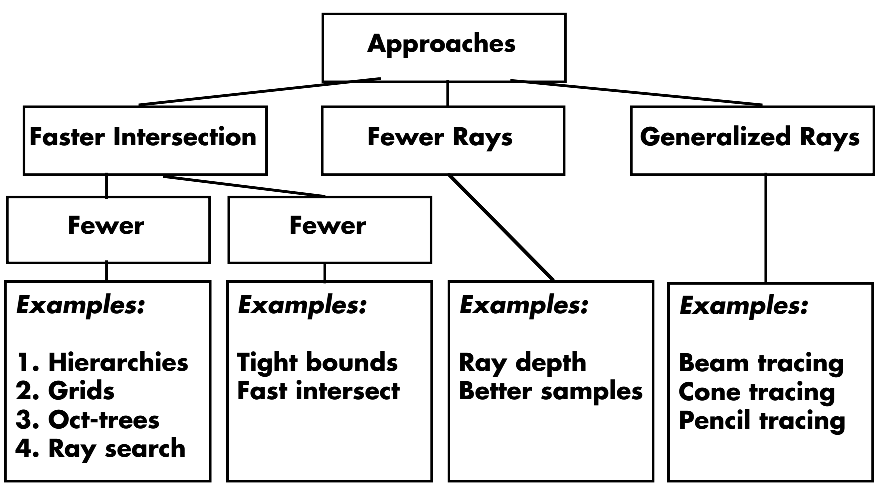
A-B
- B=0 A
- B=1 0

CSG Optimizations

2. S-Bounds (Cameroon)
   - Upward pass: form bounding volumes
   - Downward pass: propagate possible bounds
   - Repeat!



Ray Tracing Acceleration Techniques



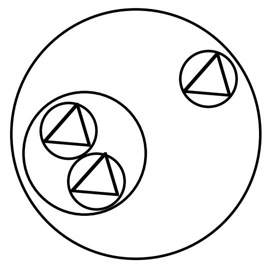
Hierarchical Bounding Volumes

Create of tree of bounding volumes

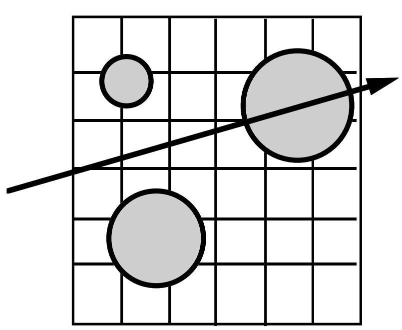
- **Children are contained within parent**
- **Creation preprocess**
  - **From model hierarchy**
  - **Automatically**

Search

```
Test parent
if intersect
   recurse on children
     in ray order
```



Grids



Fujimoto *et al.*

1. Preprocess scene
Place object in cell if overlaps cell
2. Traverse grid
3D line drawing

What resolution?

Oct-trees, k-d trees, bsp trees

Three traversal algorithms:

1. Point location from root

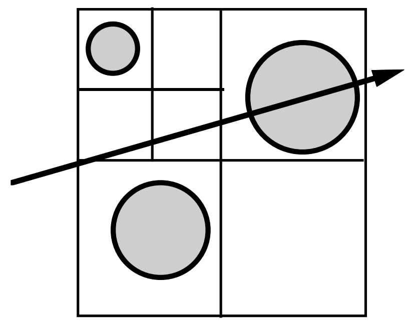
Glassner, Wyvill&Kunii

2. Inorder traversal

Arvo, Naylor, Kaplan

2. Neighbor finding

Samet



How many objects in leaves?

Mailboxes

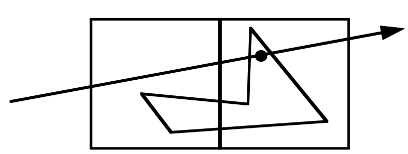
Optimize for objects that overlap multiple cells

Amanatides 1987

1. Assign each ray a number

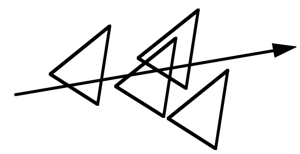
2. Object intersection cache

- **Store ray number**
- **Intersection**



Theoretical Nugget 1

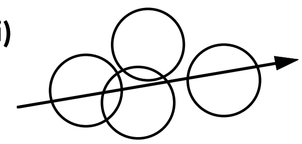
Computational Geometry of Ray Shooting



1. Triangles (Pellegrini)

Time: $O(\log n)$

Space: $O(n^{5+\varepsilon})$



2. Sphere (Guibas and Pellegrini)

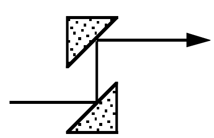
Time: $O(\log^2 n)$

Space: $O(n^{5+\varepsilon})$

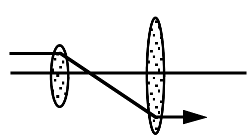
Theoretical Nugget 2

Optical Computer = Turing Machine

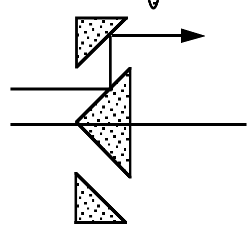
y=y+1



y=2*y



if(y>0)

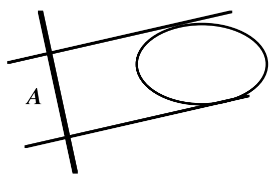


Theoretical Nugget 3

Probability that a ray in a given direction intersects a shape is equal to its projected area.

Probability that any ray intersects a shape is equal to its average projected area.

Theorem: For a convex body: $\overline{A} = \frac{S}{4}$



Sphere: