Classic Ray Tracing

Greeks: ***Do light rays proceed from the eye to the light, or from the light to the eye?***

Gauss: Rays through lenses

Appel 1968:

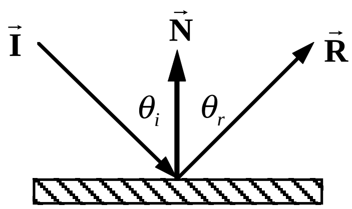
1. **Generate an image by sending one ray per pixel**
2. **Check for shadows by sending a ray to the light**

Whitted 1979: Generate images with mirror reflection and transparent refraction by recursively tracing rays

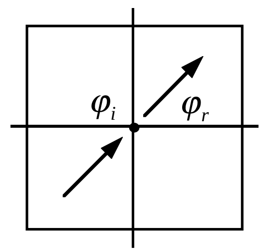
Three Ideas About Light

1. **Light rays travel in straight lines**
2. **Light rays do not interfere with each other if they cross**
3. **Light rays travel from the light sources to the eye, but the physics is invariant under path reversal (*reciprocity*).**

Law of Reflection



$$\theta_r = \theta_i$$

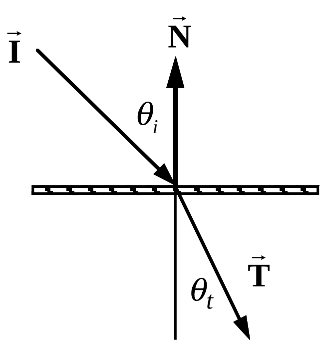


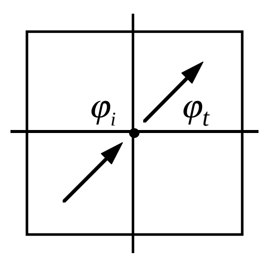
$$\varphi_r = \varphi_i \pm \pi$$

$$\vec{\mathbf{R}} + (-\vec{\mathbf{I}}) = 2\cos\theta\,\vec{\mathbf{N}} = -(\vec{\mathbf{I}} \bullet \vec{\mathbf{N}})\vec{\mathbf{N}}$$

$$\vec{\mathbf{R}} = \vec{\mathbf{I}} - (\vec{\mathbf{I}} \bullet \vec{\mathbf{N}})\vec{\mathbf{N}}$$

Snell's Law

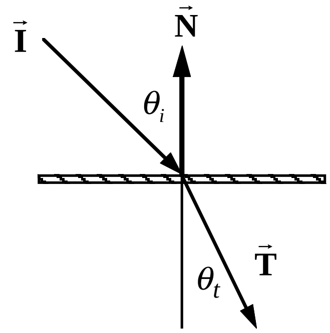




$$\varphi_t = \varphi_i \pm \pi$$

$$n_i \sin\theta_i = n_t \sin\theta_t \qquad n_i\,\vec{\mathbf{N}} \times \vec{\mathbf{T}} = n_t\,\vec{\mathbf{N}} \times \vec{\mathbf{I}}$$

Law of Refraction



$$\vec{\mathbf{N}} \times \vec{\mathbf{T}} = \mu\, \vec{\mathbf{N}} \times \vec{\mathbf{I}} \qquad \mu = n_i / n_t$$

$$\vec{\mathbf{N}} \times (\vec{\mathbf{T}} - \mu\, \vec{\mathbf{I}}) = 0 \qquad \vec{\mathbf{T}} = \mu\, \vec{\mathbf{I}} + \gamma \vec{\mathbf{N}}$$

$$\vec{\mathbf{T}}^2 = 1 = \mu^2 + \gamma^2 + 2\mu\gamma\, \vec{\mathbf{I}} \bullet \vec{\mathbf{N}}$$

$$\gamma = -\mu\, \vec{\mathbf{I}} \bullet \vec{\mathbf{N}} \pm \left\{ 1 - \mu^2 (1 - \left(\vec{\mathbf{I}} \bullet \vec{\mathbf{N}} \right)^2) \right\}^{1/2}$$

$$= \mu \cos\theta_i \pm \left\{ 1 - \mu^2 \sin^2 \theta_i \right\}^{1/2}$$

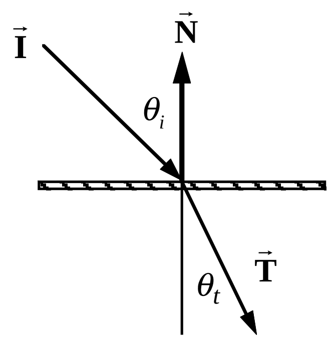
$$= \mu \cos\theta_i \pm \cos\theta_t$$

$$= \mu \cos\theta_i - \cos\theta_t \qquad \leftarrow \gamma = \mu - 1$$

Total internal reflection:

$$1 - \mu^2 (1 - \left(\vec{\mathbf{I}} \bullet \vec{\mathbf{N}} \right)^2) < 0$$

Law of Refraction



$$\vec{\mathbf{N}} \times \vec{\mathbf{T}} = \mu\, \vec{\mathbf{N}} \times \vec{\mathbf{I}} \qquad \mu = n_i / n_t$$

Total internal reflection:

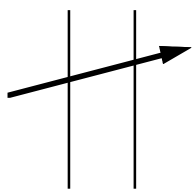
Ray-Plane Intersection

Ray: $\vec{\mathbf{P}} = \vec{\mathbf{O}} + t\vec{\mathbf{D}}$

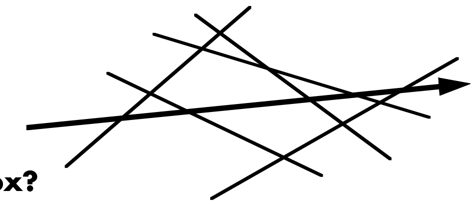
Plane: $\vec{\mathbf{P}} \bullet \vec{\mathbf{N}} + d = 0$

$$ax + by + cz + d = 0$$

Ray-Slab



Ray-ConvexPolyhedra

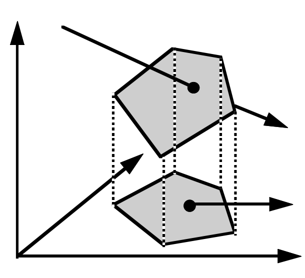


Ray-Box?

Ray-Zonahedra?

Ray-Polygon Intersection

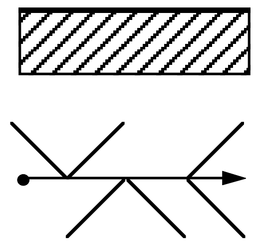
1. Find intersection with plane of support
2. Test whether point is inside 3d polygon
   a. Project onto xy plane (actually use max N coord.)
   b. Test whether point is inside 2d polygon



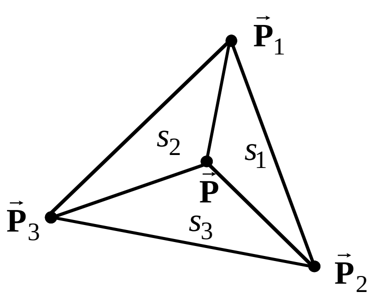
Point in Polygon

```
inside(vert v[], int n, float x, float y)
{
  int cross=0; float x0, y0, x1, y1;

  x0 = v[n-1].x - x;
  y0 = v[n-1].y - y;
  while( n-- ) {
    x1 = v->x - x;
    y1 = v->y - y;
    if( y0 > 0 ) {
      if( y1 <= 0 )
        if( x1*y0 > y1*x0 ) cross++;
    }
    else {
      if( y1 > 0 )
        if( x0*y1 > y0*x1 ) cross++;
    }
    x0 = x1; y0 = y1; v++;
  }
  return cross & 1;
}
```



Ray-Triangle Intersection



Barycentric coordinates

$$\vec{\mathbf{P}} = s_1\vec{\mathbf{P}}_1 + s_2\vec{\mathbf{P}}_2 + s_3\vec{\mathbf{P}}_3$$

$$s_1 + s_2 + s_3 = 1$$

Inside criteria

$$0 \le s_1 \le 1$$
$$0 \le s_2 \le 1$$
$$0 \le s_3 \le 1$$

Ray-Triangle Intersection

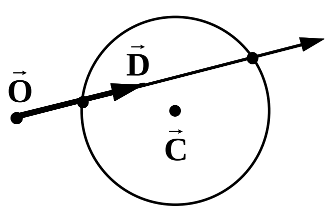
$$\vec{\mathbf{P}} = s_1\vec{\mathbf{P}}_1 + s_2\vec{\mathbf{P}}_2 + s_3\vec{\mathbf{P}}_3 \quad \Rightarrow \quad \begin{bmatrix}\mathbf{P}_1 & \mathbf{P}_2 & \mathbf{P}_3\end{bmatrix}\begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix} = [\mathbf{P}]$$

$$s_1 = \frac{\left|\mathbf{P} \quad \mathbf{P}_2 \quad \mathbf{P}_3\right|}{\left|\mathbf{P}_1 \quad \mathbf{P}_2 \quad \mathbf{P}_3\right|} = \mathbf{P} \bullet \frac{\mathbf{P}_2 \times \mathbf{P}_3}{\Delta}$$

$$s_2 = \frac{\left|\mathbf{P}_1 \quad \mathbf{P} \quad \mathbf{P}_3\right|}{\left|\mathbf{P}_1 \quad \mathbf{P}_2 \quad \mathbf{P}_3\right|} = \mathbf{P} \bullet \frac{\mathbf{P}_3 \times \mathbf{P}_1}{\Delta}$$

$$s_3 = \frac{\left|\mathbf{P}_1 \quad \mathbf{P}_2 \quad \mathbf{P}\right|}{\left|\mathbf{P}_1 \quad \mathbf{P}_2 \quad \mathbf{P}_3\right|} = \mathbf{P} \bullet \frac{\mathbf{P}_1 \times \mathbf{P}_2}{\Delta}$$

Ray-Sphere Intersection



Ray: $\vec{\mathbf{P}} = \vec{\mathbf{O}} + t\vec{\mathbf{D}}$

Sphere: $\left(\vec{\mathbf{P}} - \vec{\mathbf{C}}\right)^2 - R^2 = 0$

$$\left(\vec{\mathbf{O}} - t\vec{\mathbf{D}} - \vec{\mathbf{C}}\right)^2 - R^2 = 0$$

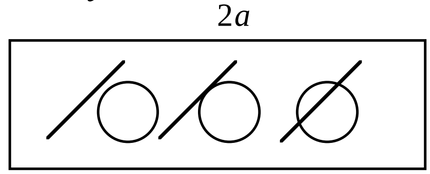
$$at^2 + bt + c = 0$$

$$a = \vec{\mathbf{D}}^2 = 1$$

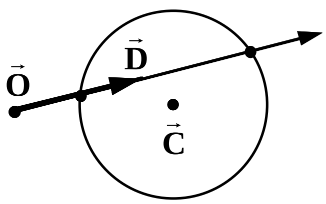
$$b = 2(\vec{\mathbf{O}} - \vec{\mathbf{C}}) \bullet \vec{\mathbf{D}}$$

$$c = (\vec{\mathbf{O}} - \vec{\mathbf{C}})^2 - R^2$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

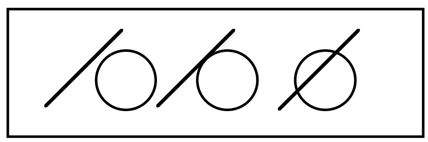


Ray-Sphere Intersection



Ray: $\vec{\mathbf{P}} = \vec{\mathbf{O}} + t\vec{\mathbf{D}}$

Sphere: $\left(\vec{\mathbf{P}} - \vec{\mathbf{C}}\right)^2 - R^2 = 0$



Ray-Implicit Surface Intersection

$$f(x,y,z) = 0$$

$$x = x_0 + x_1 t$$
$$y = y_0 + y_1 t$$
$$z = z_0 + z_1 t$$

$$f^*(t) = 0$$

1. Substitute ray equation

2. Find *positive, real* roots

Univariate root finding

- **Newton's method**
- ***Regula-falsi***
- **Interval methods**
- **Heuristics**

Ray-Algebraic Surface Intersection

$$p_n(x,y,z) = 0$$

$$x = x_0 + x_1 t$$
$$y = y_0 + y_1 t$$
$$z = z_0 + z_1 t$$

$$p_n^*(t) = 0$$

Degree *n*

- ***Linear*: Plane**
- ***Quadric*: Spheres, Cylinders, Cones, Paraboloids, Hyperboloids**
- ***Quartic*: Toris**
- ***18th*: Bicubic patch**

Polynomial root finding

- **Quadratic, cubic, quartic**
- **Descarte's rule of signs**
- **Sturm sequences**
- **Madsen-Reid algorithm**
- **Bezier/Bernoulli basis**

History

Polygons	**Appel '68**
Quadrics, CSG	**Goldstein and Nagel '71**
Tori	**Roth '82**
Bicubic patches	**Whitted '80, Kajiya '82**
Superquadrics	**Edwards and Barr '83**
Algebraic surfaces	**Hanrahan '82**
Swept surfaces	**Kajiya '83, van Wijk '84**
Fractals	**Kajiya '83**
Height fields	**Coquillart and Gangnet '84**
Deformations	**Barr '86**