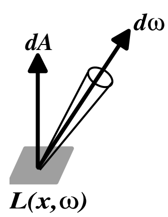
Radiance and Luminance

<u>Definition</u>: The *radiance* (*luminance*) is the power per unit projected area perpendicular to the ray per unit solid angle in the direction of the ray.



$$\boxed{\begin{aligned} d^2\Phi &= L(\vec{x},\vec{\omega})\, d\vec{\omega} \bullet d\vec{A} \\ &= L(\vec{x},\vec{\omega}) \cos\theta \, d\omega \, dA \end{aligned}}$$

$$L(x,w) \quad \left[\frac{W}{m^2 sr}\right]\left[Nit = \frac{Lumen}{m^2 sr}\right]$$

Properties of Radiance

1. Fundamental field quantity that characterizes the distribution of light in an environment.

All other quantities are derived from it.

2. Radiance invariant along a ray.

Energy and throughput conserved implies radiance conserved.

3. Response of a sensor proportional to radiance.

CS348B Lecture 7 Pat Hanrahan

Throughput

Volume element in phase space

- **Measure of lines**
- **Area-solid angle product**

$$d^2T = d\vec{\omega} \bullet d\vec{A}$$

$$T = \iint d\vec{\omega} \bullet d\vec{A}$$

Entendue $d^2E = n^2\, d^2T = n^2\, d\vec{\omega} \bullet d\vec{A}$

- **Lagrangian invariant (in a system that obeys the laws of geometrical optics)**
- **Implies L/n^2 conserved (Abbe's Law)**
- **L invariant in free space (1st Law)**
- **L does exceed maximal input**

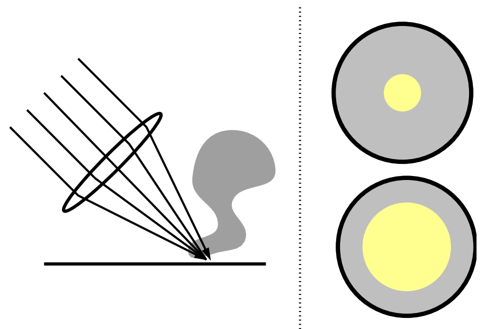
Quiz 1

Does radiance increase under a magnifying glass?

Quiz 1

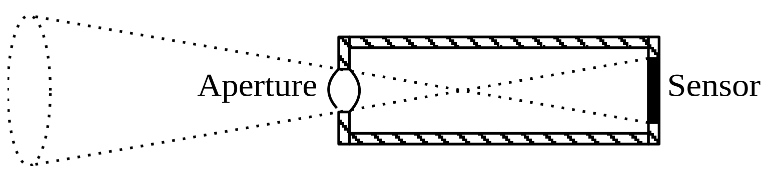
Does radiance increase under a magnifying glass?

NO!!



Radiance: 2nd Law

The response of a sensor is proportional to the radiance of the surface visible to the sensor.



$$R = \int_A \int_\Omega L \cos\theta \, d\omega \, dA = L \int_A \int_\Omega \cos\theta \, d\omega \, dA = LT$$

L ***is what should be computed and displayed.***

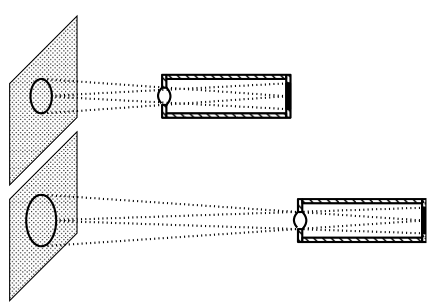
Quiz 2

Does the brightness that a wall appears to the eye depend on the distance of the viewer to the wall?

Quiz 2

Does the brightness that a wall appears to the eye depend on the distance of the viewer to the wall?

NO!!

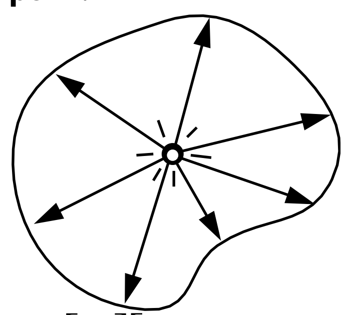


Radiant and Luminous Intensity

<u>Definition</u>: The *radiant* (*luminous*) *intensity* is the power per unit solid angle from a point.

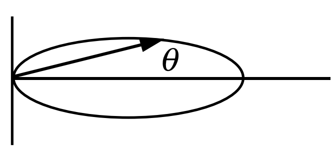
$$\frac{d\Phi}{d\omega} = I(\omega)$$

$$\Phi = \int I(\omega)\, d\omega$$



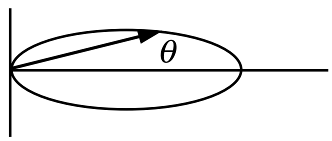
$$I(\omega) \quad \left[\frac{W}{sr}\right]\left[candela = cd = \frac{lm}{sr}\right]$$

Warn's Spotlight



$$I(\omega) = \cos^s \theta$$

Warn's Spotlight



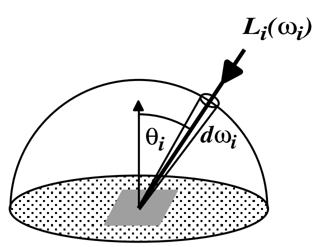
$$I(\omega) = \cos^s \theta$$

$$\Phi = \int_0^{2\pi}\int_0^1 I(\omega)\, d\cos\theta\, d\varphi = 2\pi \int_0^1 \cos^s\theta\, d\cos\theta = \frac{2\pi}{s+1}$$

$$I(\omega) = \Phi \frac{s+1}{2\pi} \cos^s \theta$$

Irradiance and Illuminance

Definition: The *irradiance* (*illuminance*) is the power per unit area incident on a surface.

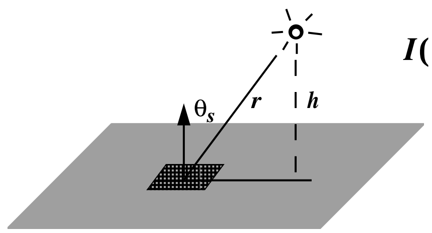


$$\boxed{E = \int_{\Omega_i} L_i \cos\theta_i \, d\omega_i}$$

$$E(x) \quad \left[\frac{W}{m^2}\right]\left[Lux = \frac{lm}{m^2}\right]$$

***This is sometimes referred to as the* radiant *and* luminous incidence.**

Isotropic Point Sources

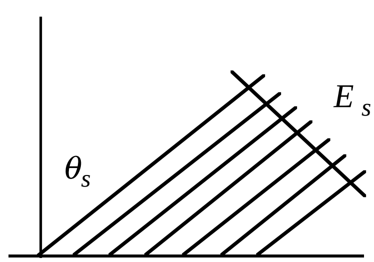


$$I(\omega) = \frac{\Phi}{4\pi}$$

$$d\Phi = E\,dA = I\,d\omega = \frac{\Phi}{4\pi}\frac{\cos\theta_s}{r^2}dA = \frac{\Phi}{4\pi}\frac{\cos^3\theta_s}{h^2}dA$$

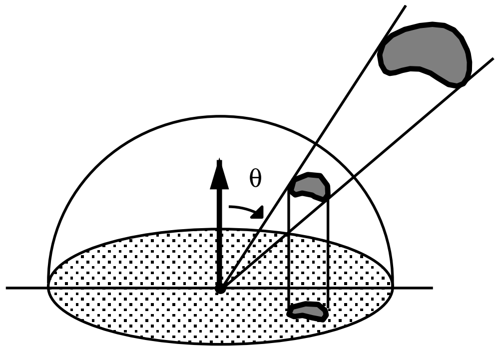
- Note inverse square law fall off.
- Note cosine dependency

Distant Source



$$E = E_s \cos\theta_s$$

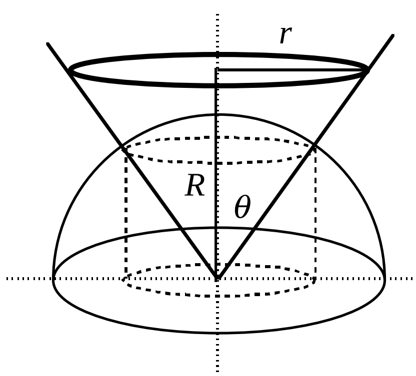
Hemisphere: Projected Solid Angle



$$\int_{\Omega} d\vec{\omega} \bullet d\hat{A} = \int_{\Omega} \cos\theta \, d\omega$$

$$\int_{H} \cos\theta \, d\omega = \pi$$

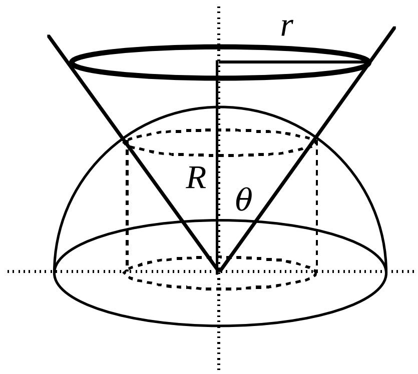
Disk



$$E = \int L \cos\theta \, d\omega$$

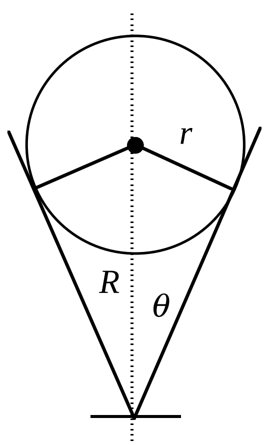
$$=$$

Disk



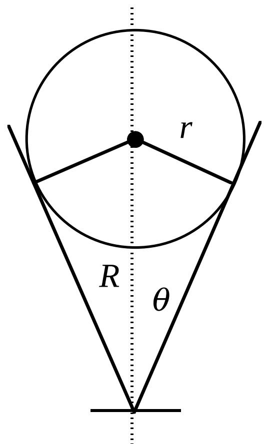
$$E = \int L \cos\theta \, d\omega$$
$$= L\pi \sin^2\theta$$
$$= L\pi \frac{r^2}{r^2 + R^2}$$

Sphere



$$E = \int L \cos\theta \, d\omega$$
$$= L\pi \sin^2\theta$$
$$=$$

Sphere



$$E = \int L \cos\theta \, d\omega$$
$$= L\pi \sin^2\theta$$
$$= L\pi \frac{r^2}{R^2}$$

The Sun

Solar constant (normal incidence at zenith)

Irradiance **1353 W/m2**

Illuminance **127.5 Lumen/m2 = 127.5 Kilo-Lux**

Solar angle

α **= .25 degrees = .004 radians(half angle)**

$\omega = \pi \sin^2 \alpha$ **= 6 x 10-5 steradians**

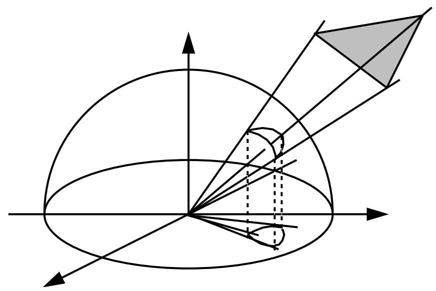
Radiance

$$L = \frac{E}{\omega} = \frac{1.353 \times 10^3 \, W/m^2}{6 \times 10^{-5} \, sr} = 2.25 \times 10^7 \,\text{Nits}$$

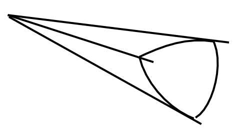
Pluto (6 tera-meters) 50 Lux - read a newspaper

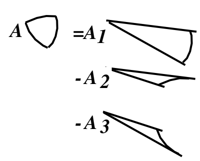
Deep space -> 20 micro-lux (see, but not read!)

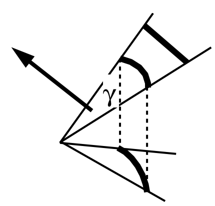
Polygonal Source (Lambert)



Polygonal Source







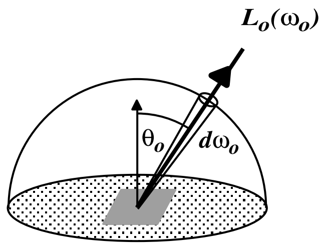
$$\int_P \cos\theta \, d\omega = \sum_{i=1}^{n} A_i$$

$$E = \sum_{i=1}^{n} \gamma_i \vec{\mathbf{N}} \bullet \vec{\mathbf{N}}_i$$

$$A_i = \gamma_i \vec{\mathbf{N}} \bullet \vec{\mathbf{N}}_i$$

Radiosity and Luminosity

Definition: The *radiosity* (*luminosity*) is the energy per unit area leaving a surface.



$$\boxed{B = \int_{\Omega_o} L_o \cos\theta_o \, d\omega_o}$$

$$B(x) \quad \left[\frac{W}{m^2}\right]\left[Lux = \frac{lm}{m^2}\right]$$

This is officially referred to as the* radiant *and* luminous *exitance.

Uniform Diffuse Source

$$\boxed{\begin{aligned} B &= \int_{\Omega_o} L \cos\theta \, d\omega \\ &= L \int_{\Omega_o} \cos\theta \, d\omega \\ &= \pi L \end{aligned}}$$

$$L = \frac{B}{\pi}$$

$$blondel = apostilb = \tfrac{1}{\pi} nit = \tfrac{1}{\pi} cd / m^2 \quad (skot = 10^{-3} apostilb)$$

$$lamberts = \tfrac{1}{\pi} cd / cm^2$$

$$foot-lamberts = \tfrac{1}{\pi} cd / ft^2 \qquad (glim = 10^{-3} foot-lambert)$$

"Thus one nit is one lux per steradian is one candela per square meter is one lumen per square meter per steradian. Got it?" *Kajiya*