Physically-Based Rendering

Basic optics

- Physics of light and color
- Geometrical optics
  - Ray "metaphor"
  - Reflection and transmission
- Radiative transfer
  - Radiometry and photometry; measurement
  - Transport theory and integral equations

Why Physically-Based Rendering?

- Interesting scientific challenge
- Necessary to simulate/augment reality
  - Applications: design, entertainment
- Use real-world data
  - Acquire geometric models
  - Acquire reflection models
- Example of high complexity simulation
  - "Grand Challenge" type problem
  - Procedural modeling and data amplification

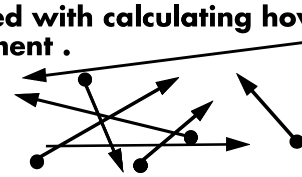
Questions

1. How is light measured?
2. How is the spatial distribution of light energy described?
3. How is reflection from a surface characterized?
4. What are the conditions for equilibrium flow of light in an environment?

What do the numbers 0-255 mean?

Transport Theory

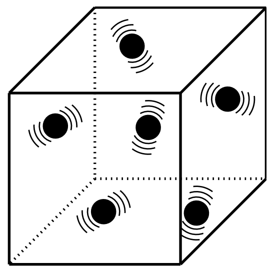
- **Transport theory is concerned with calculating how *stuff* Q flows in the environment .**
  - **Mass m**
  - **Charge q**
  - **Radiant energy Φ**



- **Transport quantities are built around a core of basic geometric ideas. These are tricky!**

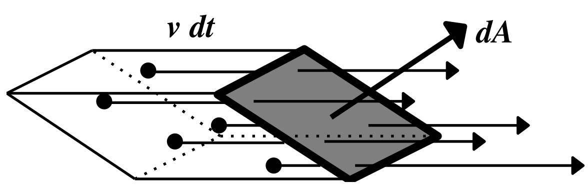
  The easiest way (for the computer graphicist) to learn transport theory is to think in terms of particles (photons).

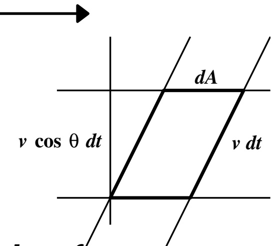
Particle Density



$$N(\vec{x},t)\, d^3x$$

Flow Across a Surface





$$Q(\vec{x}) = N(\vec{x})\, d^3x = N(\vec{x}) \cos\theta\; v dt\, dA$$
$$= N(\vec{x})(\vec{v}\, dt) \bullet d\vec{A}$$

The surface can be a real physical surface or a virtual test surface.

Flux and Flux Density

Nothing flows through 0 area in 0 time!

- **Flux [Stuff/Time] (or Rate or Flow)**

$$\Phi = \frac{dQ}{dt} = N(\vec{x})\vec{v} \bullet d\vec{A} = N(\vec{x})\, v \cos\theta\, dA$$

- **Flux density [Stuff/(Time•Area)]**

$$\frac{d\Phi}{dA} = \frac{d^2Q}{dt\, dA} = N(\vec{x})\vec{v} \bullet d\hat{A} = N(\vec{x})\, v \cos\theta$$

Phase Space

Particles characterized by position and <u>velocity</u>

$$\boxed{d^3x} \times \boxed{d^3v} \qquad N(\vec{x}) = \int n(\vec{x},\vec{v})\, d^3v$$

- **Phase space or angular flux**

$$\varphi(\vec{x},\vec{v}) = n(\vec{x},\vec{v})\, v$$

- **Phase space or angular current density**

$$\vec{j}(\vec{x},\vec{v}) = n(\vec{x},\vec{v})\,\vec{v}$$

Angles and Solid Angles

- **Angles**

  $\theta = (length)/(radius)$

  $\Rightarrow$ **2π radians in a circle**

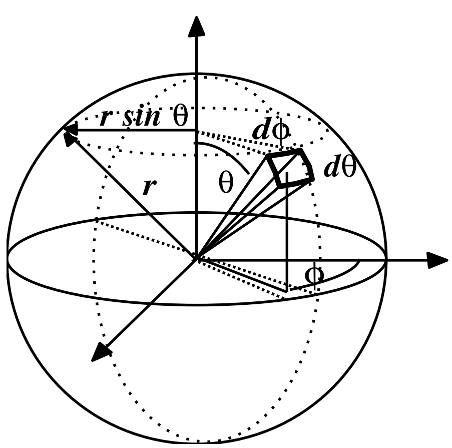
- **Solid angles**

  $\omega = (area)/(radius\ squared)$

  $\Rightarrow$ **4π steradians in a sphere**

  If the area is not on the sphere, then the solid angle subtended by the area is equal to the area projected onto the unit sphere.

Differential Solid Angles



$$dA = (r\,d\theta)(r\sin\theta\,d\phi) = r^2\sin\theta\,d\theta\,d\phi$$

$$d\omega = dA/r^2 = \sin\theta\,d\theta\,d\phi$$

Velocity Distributions

Velocity

$$n(\vec{x},\vec{v})\,d^3v = n(\vec{x},\vec{\omega},v)\,v^2 d\omega\,dv$$

Energy

$$n(\vec{x},\vec{v})\,d^3v = n(\vec{x},\vec{\omega},E)\,d\omega\,dE$$

Frequency/Wavelength

From Photons to Light

Particles to light energy

- Carry $hf=hc/\lambda$ energy per particle
- Do not interact with each other
- All travel with the same speed, but in different directions
- Characterize distribution of directions by number traveling in a differential solid angle

Radiance $$\boxed{L(\vec{x},\vec{\omega},\lambda) = \frac{hc}{\lambda}\,\varphi(\vec{x},\vec{\omega},\lambda)}$$

Radiometry vs. Photometry

- **Radiance and Radiometry [Units = Watts]**

  Physical measurement of electromagnetic energy.

- **Luminance and Photometry [Units = Lumen]**

  Perceptual measurement of relative subjective sensation due to light of different wavelengths.

- **Brightness [Units = Brils]**

  Perceptual measurement of the relative perceived sensation of light of different intensities.

Bouguer's Photometry

Photometry

Definition of a standard candle

- **Standard Bouguer candle**
- **Hefner gas lamp**
- **1909-1948 international candle**
  Carbon filament vacumn lamp
- **1948-1979 candela**
  1/600,000 sq. meter blackbody at freezing point of platinum
- **1979**
  540 x 10^{12} Hz (550nm) with 1/683 W/sr

Radiant Energy and Power

Power: Watts vs. Lumen

- **Energy efficiency**
- **Spectral efficacy**

Energy: Joules vs. Talbot

- **Since the velocity of light is so fast, may typically ignore distinction between power and energy**
- **Exposure - Reaction rates**
  - **Film response**
  - **Skin - sunburn**

Radiometric and Photometric Terms

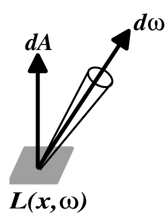
Physics	Radiometry	Photometry
Energy	Radiant Energy	Luminous Energy
Flux (Power)	Radiant Power	Luminous Power
Flux Density	Irradiance Radiosity	Illuminance Luminosity
Angular Flux Density	Radiance	Luminance
Intensity	Radiant Intensity	Luminous Intensity

Photometric Units

Photometry	Units		
	MKS	CGS	British
Luminous Energy	Talbot		
Luminous Power	Lumen		
Illuminance Luminosity	Lux	Phot	Footcandle
Luminance	Nit Apostilb, Blondel	Stilb Lambert	Footlambert
Luminous Intensity	Candela (Candle, Candlepower, Carcel, Hefner)		

Radiance and Luminance

<u>Definition</u>: The *radiance* (*luminance*) is the power per unit projected area perpendicular to the ray per unit solid angle in the direction of the ray.



$$\boxed{\begin{aligned} d^2\Phi &= L(\vec{x},\vec{\omega})\, d\vec{\omega} \bullet d\vec{A} \\ &= L(\vec{x},\vec{\omega}) \cos\theta \, d\omega \, dA \end{aligned}}$$

Properties of Radiance

1. Fundamental field quantity that characterizes the distribution of light in an environment.

   All other quantities are derived from it.

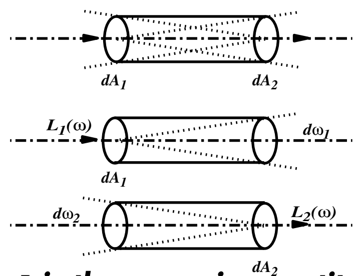
2. Radiance invariant along a ray.

   Energy and throughput conserved implies radiance conserved.

3. Response of a sensor proportional to radiance.

Radiance: 1st Law

The radiance in the direction of a light ray remains constant as the ray propagates.



$$d\Phi_1 = L_1\, d\omega_1\, dA_1 = L_2\, d\omega_2\, dA_2 = d\Phi_2$$

$$d\omega_1 = dA_2 / r^2$$
$$d\omega_2 = dA_1 / r^2$$
$$d\omega_1\, dA_1 = \frac{dA_1\, dA_2}{r^2} = d\omega_2\, dA_2$$

$$\therefore L_1 = L_2$$

L is the numeric quantity that should be associated with rays in ray tracers.

Throughput

Volume element in phase space

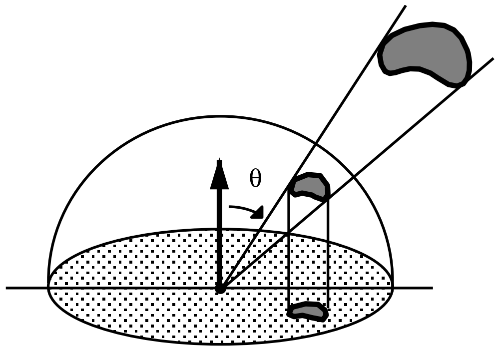
- **Measure of lines**
- **Area-solid angle product**

$$d^2T = d\vec{\omega} \bullet d\vec{A}$$
$$T = \iint d\vec{\omega} \bullet d\vec{A}$$

Entendue $d^2E = n^2\, d^2T = n^2\, d\vec{\omega} \bullet d\vec{A}$

- **Lagrangian invariant (in a system that obeys the laws of geometrical optics)**
- **Implies L/n^2 conserved (Abbe's Law)**
- **L invariant in free space (1st Law)**
- **L does exceed maximal input**

Projected Solid Angle



$$\int_{\Omega} \cos\theta \, d\omega = \pi$$