CS 348B
Winter '95
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Febraury 9, 1995
Handout #4

Written Assignment 1

Due Date: 11:59pm, Febraury 21

1. An important photometric constant is the *solar constant*, which is equal to the irradiance (1353 W/m^2) outside the earth's atmosphere due to radiation from the sun. Less important perhaps is the lunar constant, that is the irradiance entering the earth's atmosphere due to reflected light from the moon. The lunar constant is roughly equal to 10^{-3} W/m^2. Given that the moon has a diameter of 2163 miles, and that the average distance between the center of the earth and the center of the moon is 238,840 miles, compute the radiance emanating from the moon's surface.

2. Consider a hemispherical sky above a surface patch.

A. Suppose the surface patch is tilted and makes an angle of $\alpha < \pi/2$ with respect to the zenith. Because of the tilt, only part of the sky is visible to the surface. Assume the radiance of the sky is uniform and equal to L. Show that the irradiance on the tilted surface due to a uniform sky is $E = L\pi cos^2\frac{\alpha}{2}$.

B. The radiance of a cloudy sky is non-uniform and may be approximated by a Cardoid distribution; that is,

$$L(\theta) = Lcos\theta$$

Here θ is zero in the direction of the zenith. Thus, this radiance distribution is maximal straight up and zero at the horizon. Compute the irradiance from the cloudy sky (of an untilted patch).

3. An important consequence of the laws of geometrical optics is that throughput of an optical system is conserved. To show this in general is difficult, however, in this problem we will compute the throughput before and after refraction across a planar surface.

Suppose an infinitesimal beam of light with cross-sectional solid angle $d\omega_i$ impinges upon a surface with an angle of incidence equal to θ_i. This beams is refracted across the surface according to Snell's Law.

$$n_t \sin\theta_t = n_i \sin\theta_i$$

where θ_t is the angle of the refracted beam with respect to the surface normal. Show that

$$d\omega_t = \frac{n_i^2 \cos\theta_i}{n_t^2 \cos\theta_t} d\omega_i$$

or,

$$n_t^2 \cos\theta_t \, d\omega_t = n_i^2 \cos\theta_i \, d\omega_i$$

This proves that under refraction, the quantity $n^2 \cos\theta \, d\omega dA$ remains unchanged.