Camera Simulation

<table>
<thead>
<tr>
<th>Effect</th>
<th>Cause</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field of view</td>
<td>Film size, stops and pupils</td>
</tr>
<tr>
<td>Depth of field</td>
<td>Aperture, focal length</td>
</tr>
<tr>
<td>Motion blur</td>
<td>Shutter</td>
</tr>
<tr>
<td>Exposure</td>
<td>Film speed, aperture, shutter</td>
</tr>
</tbody>
</table>

References

Photography, B. London and J. Upton
Optics in Photography, R. Kingslake
The Camera, The Negative, The Print, A. Adams

Topics

Ray tracing lenses
Focus
Field of view
Depth of focus / depth of field
Exposure
Lenses

Refraction

Snell’s Law

\[ n' \sin I' = n \sin I \]
Paraxial Approximation

\[ \sin U \approx u \]
\[ \tan U \approx u \]

Rays deviate only slightly from the axis

Incident Ray

Angles: ccw is positive; cw is negative

\[ I = U - \phi \]

The sum of the interior angles is equal to the exterior angle.
Refracted Ray

\[ I' = U' - \phi \]

Derivation

Paraxial approximation

\[
\begin{align*}
I &= U - \phi \quad \Rightarrow \quad i = u - \phi \\
I' &= U' - \phi \quad \Rightarrow \quad i' = u' - \phi
\end{align*}
\]
Derivation

Paraxial approximation

\[ I = U - \phi \Rightarrow i = u - \phi \]
\[ I' = U' - \phi \Rightarrow i' = u' - \phi \]

Snell’s Law

\[ n' \sin I' = n \sin I \Rightarrow n'i' = ni \]
\[ n'(u' - \phi) = n(u - \phi) \]

Ray Coordinates

\[ u = \frac{h}{-z} \quad -u' = \frac{h}{z'} \quad -\phi = \frac{h}{R} \]
**Gauss’ Formula**

Paraxial approximation to Snell’s Law

\[ n'(u' - \phi) = n(u - \phi) \]

Ray coordinates

\[ u' = -\frac{h}{z'} \quad \phi = -\frac{h}{R} \quad u = -\frac{h}{z} \]

Thin lens equation

\[ n'\left(\frac{h}{z'} - \frac{h}{R}\right) = n\left(\frac{h}{z} - \frac{h}{R}\right) \]

\[ \frac{n'}{z'} = \frac{n}{z} + \frac{(n' - n)}{R} \quad \text{Holds for any height, any ray!} \]

---

**Vergence**

\[ V < 0 \quad V = 0 \quad V > 0 \]

Vergence

\[ V = \frac{n}{r} \approx \frac{n}{z} \quad \left[ \frac{1}{m} = \text{diopters} \right] \]

Thin lens equation

\[ V' = V + P \]

Surface Power equation

\[ P = (n' - n)\frac{1}{R} \]
Lens-makers Formula

Refractive Power

\[ P = (n' - n) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) = \frac{1}{f} \]

Converging

Diverging

Conjugate Points

\[ \frac{1}{z'} = \frac{1}{z} + \frac{1}{f} \]

To focus: move lens relative to backplane
Horizontal rays converge on focal point in the focal plane
Gauss’ Ray Tracing Construction

Parallel Ray
Focal Ray
Chief Ray

Object Image

Ray Tracing: Finite Aperture

Focal Plane Aperture Plane Back Plane
Real Lens

Cutaway section of a Vivitar Series 1 90mm f/2.5 lens
Cover photo, Kingslake, *Optics in Photography*

Double Gauss

Data from W. Smith,
*Modern Lens Design*, p 312

<table>
<thead>
<tr>
<th>Radius  (mm)</th>
<th>Thick (mm)</th>
<th>$n_d$</th>
<th>V-no</th>
<th>aperture</th>
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</tbody>
</table>
Ray Tracing Through Lenses

From Kolb, Mitchell and Hanrahan (1995)

Thick Lenses

Equivalent Lens

Refraction occurs at the principal planes
Field of View

From London and Upton
Field of View

Field of view

\[ \tan \frac{\text{fov}}{2} = \frac{\text{filmsize}}{f} \]

Types of lenses

- **Normal** 26°
  Film diagonal ~ focal length
- **Wide-angle** 75-90°
- **Narrow-angle** 10°

Redrawn from Kingslake, *Optics in Photography*
Perspective Transformation

Thin lens equation

\[
\frac{1}{z'} = \frac{1}{z} + \frac{1}{f} \implies z' = \frac{fz}{z + f}
\]

\[
\implies x' = \frac{fx}{z + f}
\]

\[
\implies y' = \frac{fy}{z + f}
\]

Represent transformation as a 4x4 matrix

Depth of Field
Depth of Field

From London and Upton

Circle of Confusion

Circle of confusion proportional to the size of the aperture

\[ \frac{c}{a} = \frac{d'}{z'} = \frac{s' - z'}{z'} \]
Depth of Focus [Image Space]

Depth of focus ≡

Equal circles of confusion

Two planes: near and far

\[
\frac{c}{a} \frac{d_n'}{z_n'} = \frac{s'-z_n'}{z_n'} = \frac{1}{z_n'} = \frac{1}{s'} \left(1 + \frac{c}{a}\right)
\]

\[
\frac{c}{a} \frac{d_f'}{z_f'} = \frac{s'-z_f'}{z_f'} = \frac{1}{z_f'} = \frac{1}{s'} \left(1 - \frac{c}{a}\right)
\]
Depth of Focus [Image Space]

Depth of focus ≡
Equal circles of confusion

\[
\frac{1}{z'_f} = \frac{1}{s'} \left( 1 + \frac{c}{a} \right) \quad \frac{1}{z'_n} = \frac{1}{s'} \left( 1 - \frac{c}{a} \right)
\]

\[
\frac{1}{z'_f} + \frac{1}{z'_n} = 2 \frac{1}{s'}
\]

\[
\frac{1}{z'_f} - \frac{1}{z'_n} = \frac{2c}{a} \frac{1}{s'}
\]

Depth of Field [Object Space]

Depth of field ≡
Equal circles of confusion

\[
\frac{1}{s'} = \frac{1}{s} \frac{1}{f} \quad \frac{1}{z'_n} = \frac{1}{z_n} + \frac{1}{f} \quad \frac{1}{z'_f} = \frac{1}{z_f} + \frac{1}{f}
\]

\[
\frac{1}{z_n} + \frac{1}{z_f} = 2 \frac{1}{s}
\]

\[
\frac{1}{z_n} - \frac{1}{z_f} = \frac{2c}{a} \left( \frac{1}{f} - \frac{1}{s} \right) \approx \frac{2c}{a} \frac{1}{f}
\]
**Hyperfocal Distance**

\[
\frac{1}{z_n} - \frac{1}{z_f} = \frac{2c}{a} \frac{1}{f^2} = 2 \cdot \frac{cN}{f^2} \equiv \frac{2}{H}
\]

When

\[s \to H \Rightarrow z_n = \frac{H}{2}, z_f = \infty\]

**H is the hyperfocal distance**

**Depth of Field Scale**

Reciprocal of Distance

<table>
<thead>
<tr>
<th>0.0</th>
<th>0.01</th>
<th>0.02</th>
<th>0.03</th>
<th>0.04</th>
<th>0.05</th>
<th>0.06</th>
<th>0.07</th>
<th>0.08</th>
</tr>
</thead>
<tbody>
<tr>
<td>∞</td>
<td>100</td>
<td>50</td>
<td>25</td>
<td>20</td>
<td>12.5</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Factors Affecting DOF


\[
\frac{1}{H} = \frac{cN}{f^2}
\]

Resolving Power

- Diffraction limit
  \[ c = 1.22 \frac{f}{\lambda} \quad [= 1.22 \times 64 \times .500 \mu m = 0.040 \text{ mm}] \]
- 35mm film (Leica standard)
  \[ c = 0.025 \text{ mm} \]
- CCD/CMOS pixel aperture
  \[ c = 0.0116 \text{ mm (Nikon D1)} \]
Exposure

Image Irradiance

\[ E = \int_{\Omega} L \cos \theta d\omega = L \pi \sin^2 \theta = L \frac{\pi}{4} \left( \frac{a}{f} \right)^2 \]
Relative Aperture or F-Stop

\[ a = \frac{f}{N} \]

F-Number and exposure: 

\[ E = L \frac{\pi}{4} \frac{1}{N^2} \]

F-stops: 1.4 2 2.8 4.0 5.6 8 11 16 22 32 45 64
1 stop doubles exposure

Camera Exposure

Exposure \( H = E \times T \)

Exposure overdetermined

- Aperture: f-stop - 1 stop doubles \( H \)
  - Decreases depth of field
- Shutter: Doubling the open time doubles \( H \)
  - Increases motion blur
Aperture vs Shutter

From London and Upton

f/16 1/8s  
f/4 1/125s  
f/2 1/500s

High Dynamic Range

Sixteen photographs of the Stanford Memorial Church taken at 1-stop increments from 30s to 1/1000s.

From Debevec and Malik, High dynamic range photographs.
Simulated Photograph

Adaptive histogram
With glare, contrast, blur

Camera Simulation

\[ R = \iiint L(x', \omega', t, \lambda) S(x', \omega', t) L(T(x', \omega', \lambda), t, \lambda) d\tilde{A}(x') \cdot d\tilde{\omega}' dt d\lambda \]

**Sensor response**  \[ P(x', \lambda) \]

**Lens**  \[ (x, \omega) = T(x', \omega', \lambda) \]

**Shutter**  \[ S(x', \omega', t) \]

**Scene radiance**  \[ L(x, \omega, t, \lambda) \]