The Light Field

Light field = radiance function on rays
Conservation of radiance
Measurement equation
Throughput and counting rays
Conservation of throughput
Area sources and irradiance
Form factors and radiosity

From London and Upton

Light Field = Radiance(Ray)
**Field Radiance**

**Definition:** The field *radiance* (*luminance*) at a point in space in a given direction is the power per unit solid angle per unit area perpendicular to the direction

\[ L(x, \omega) \]

Radiance is the quantity associated with a ray

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**Light Probe ⇒ Environment Map**

\[ L(x, y, z, \theta, \varphi) \]

*Miller and Hoffman, 1984*
Properties of Radiance

1. Fundamental field quantity that characterizes the distribution of light in an environment.
   - Radiance is a function on rays
   - All other field quantities are derived from it
2. Radiance invariant along a ray.
   - 5D ray space reduces to 4D
3. Response of a sensor proportional to radiance.
1st Law: Conservation of Radiance

The radiance in the direction of a light ray remains constant as the ray propagates.

\[ \frac{d^2 \Phi_1}{dA_1} \frac{d \omega_1}{d \omega_1} = \frac{d^2 \Phi_2}{dA_2} \frac{d \omega_2}{d \omega_2} \]

\[ d^2 \Phi_1 = L_1 d \omega_1 dA_1 \]

\[ d^2 \Phi_2 = L_2 d \omega_2 dA_2 \]

\[ d \omega_1 dA_1 = \frac{dA_1 dA_2}{r^2} = d \omega_2 dA_2 \]

\[ \therefore L_1 = L_2 \]

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Spherical Gantry \( \Rightarrow \) 4D Light Field

Capture all the light leaving an object - like a hologram.

\[ L(x, y, \theta, \phi) \]

\[ (\theta, \phi) \]
Two-Plane Light Field

$$L(u, v, s, t)$$

Multi-Camera Array $\Rightarrow$ Light Field
Throughput Counts Rays

Define an infinitesimal beam as the set of rays intersecting two infinitesimal surface elements

\[ r(u_1, v_1, u_2, v_2) \]

\[ dA_1(u_1, v_1) \quad \quad dA_2(u_2, v_2) \]

\[ d^2T = \frac{dA_1dA_2}{|x_1 - x_2|^2} \]

\( T \) measures/count the number of rays in the beam

Conservation of Throughput

- Throughput conserved during propagation
  - Number of rays conserved
  - Assuming no attenuation or scattering
- \( n^2 \) (index of refraction) times throughput invariant under the laws of geometric optics
  - Reflection at an interface
  - Refraction at an interface
    - Causes rays to bend (kink)
  - Continuously varying index of refraction
    - Causes rays to curve; mirages
Conservation of Radiance

Radiance is the ratio of two quantities:
1. Power
2. Throughput

\[ L(r) = \lim_{\Delta T \to 0} \frac{\Delta \Phi(\Delta T)}{\Delta T} = \frac{d\Phi}{dT} \]

Since power and throughput are conserved,
∴ Radiance conserved

Quiz

Does radiance increase under a magnifying glass?

No!!
Radiance: 2nd Law

The response of a sensor is proportional to the radiance of the surface visible to the sensor.

\[ R = \int \int_{A \Omega} L d\omega dA = \bar{L}T \quad T = \int \int_{A \Omega} d\omega dA \]

\( L \) is what should be computed and displayed.
\( T \) quantifies the gathering power of the device; the higher the throughput the greater the amount of light gathered.

Quiz

Does the brightness that a wall appears to the sensor depend on the distance?

No!!
Measuring Rays = Throughput

Throughput Counts Rays

Define an infinitesimal beam as the set of rays intersecting two infinitesimal surface elements

\[ r(u_1, v_1, u_2, v_2) \]

\[ dA_1(u_1, v_1) \quad - \quad dA_2(u_2, v_2) \]

\[ d^2T = \frac{dA_1 dA_2}{|x_1 - x_2|^2} \]

Measure/count the number of rays in the beam
Parameterizing Rays

Parameterize rays wrt to receiver \( r(u_2, v_2, \theta_2, \phi_2) \)

\[
d\omega_2(\theta_2, \phi_2) \quad d^2T = \frac{dA_1}{|x_1 - x_2|^2} dA_2 = d\omega_2 dA_2
\]

Parameterizing Rays

Parameterize rays wrt to source \( r(u_1, v_1, \theta_1, \phi_1) \)

\[
dA_1(u_1, v_1) \quad d^2T = dA_1 \frac{dA_2}{|x_1 - x_2|^2} = dA_1 d\omega_1
\]
Parameterizing Rays

Tilting the surfaces reparameterizes the rays

\[ r(u_1, v_1, u_2, v_2) \]
\[ dA_1(u_1, v_1) \quad dA_2(u_2, v_2) \]

\[ d^2T = \frac{\cos \theta_1 \cos \theta_2}{|x_1 - x_2|^2} \, dA_1 \, dA_2 \]
\[ = d\vec{\omega}_1 \cdot d\vec{A}_1 \]
\[ = d\vec{\omega}_2 \cdot d\vec{A}_2 \]

All these throughputs must be equal.

Parameterizing Rays: \( S^2 \times \mathbb{R}^2 \)

Parameterize rays by \( r(x, y, \theta, \phi) \)

Projected area \( \tilde{A}(\vec{\omega}) \)

Measuring the number or rays that hit a shape

\[ T = \int_{S^2} d\omega(\theta, \phi) \int_{\mathbb{R}^2} dA(x, y) \]
\[ = \int_{S^2} \tilde{A}(\theta, \phi) \, d\omega(\theta, \phi) \]
\[ = 4\pi \bar{A} \]

Sphere:

\[ T = 4\pi \bar{A} = 4\pi^2 R^2 \]
Parameterizing Rays: $M^2 \times S^2$

Parameterize rays by $r(u,v,\theta,\phi)$

\[
T = \int_{M^2} dA(u,v) \int_{H^2(N)} \cos \theta \, d\omega(\theta,\phi)
\]

**Sphere:** $T = \pi S = 4\pi^2 R^2$

**Crofton’s Theorem:** $4\pi A = \pi S \Rightarrow A = \frac{S}{4}$

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**Incident Surface Radiance**

**Definition:** The incoming surface radiance (luminance) is the power per unit solid angle per unit projected area arriving at a receiving surface

\[
L_i(x,\omega) = \frac{d^2\Phi_i(x,\omega)}{d\omega \cdot dA}
\]
Exitant Surface Radiance

Definition: The outgoing surface radiance (luminance) is the power per unit solid angle per unit projected area leaving at surface

\[ L_o(x, \omega) \equiv \frac{d^2 \Phi_o(x, \omega)}{d\omega \cdot d\tilde{A}} \]

Alternatively: the intensity per unit projected area leaving a surface

Irradiance from a Uniform Area Source
Irradiance from the Environment

\[ d^2 \Phi_i(x, \omega) = L_i(x, \omega) \cos \theta \, dA \, d\omega \]

\[ dE(x, \omega) = L_i(x, \omega) \cos \theta \, d\omega \]

\[ E(x) = \int_{H^2} L_i(x, \omega) \cos \theta \, d\omega \]

Uniform Area Source

\[ E(x) = \int_{H^2} L \cos \theta \, d\omega \]

\[ = \int_{\Omega} \cos \theta \, d\omega \]

\[ = L \tilde{\Omega} \]
Projected Solid Angle

\[ \int_{H^2} \cos \theta \, d\omega = \pi \]

Uniform Disk Source

**Geometric Derivation**

\[ \tilde{\Omega} = \pi \sin^2 \alpha \]

**Algebraic Derivation**

\[
\tilde{\Omega} = \int_{1}^{\cos \alpha} \int_{0}^{2\pi} \cos \theta \, d\phi \, d\cos \theta \\
= 2\pi \cos \alpha \frac{\cos \theta}{2} \bigg|_{1}^{\cos \alpha} \\
= \pi \sin^2 \alpha \\
= \pi \frac{r^2}{r^2 + h^2}
\]
Spherical Source

**Geometric Derivation**

\[ \tilde{\Omega} = \pi \sin^2 \alpha \]

**Algebraic Derivation**

\[ \tilde{\Omega} = \int \cos \theta \, d\omega \]
\[ = \pi \sin^2 \alpha \]
\[ = \pi \frac{r^2}{R^2} \]

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The Sun

**Solar constant (normal incidence at zenith)**

- **Irradiance** 1353 W/m²
- **Illuminance** 127,500 lm/m² = 127.5 kilolux

**Solar angle**

\[ \alpha = .25 \text{ degrees} = .004 \text{ radians (half angle)} \]
\[ \tilde{\Omega} = \pi \sin^2 \alpha \approx \pi \alpha^2 = 6 \times 10^{-5} \text{ steradians} \]

**Solar radiance**

\[ L = \frac{E}{\tilde{\Omega}} = \frac{1.353 \times 10^3 \text{ W/m}^2}{6 \times 10^{-5} \text{ sr}} = 2.25 \times 10^7 \frac{\text{W}}{\text{m}^2 \cdot \text{sr}} \]
Polygonal Source
Polygonal Source

Lambert’s Formula

\[ A_i = \gamma_i \vec{N}_i \cdot \vec{N} \]

\[ \sum_{i=1}^{3} A_i = A_1 - A_2 - A_3 \]

\[ \sum_{i=1}^{n} A_i = \sum_{i=1}^{n} \gamma_i \vec{N}_i \cdot \vec{N} \]
Penumbras and Umbras

Form Factors
Types of Throughput

1. Infinitesimal beam of rays (radiance)
   \[ d^2T(dA, dA') \equiv \frac{\cos \theta \cos \vartheta}{|x-x'|^2} dA(x) dA(x') \]

2. Infinitesimal-finite beam (irradiance calc.)
   \[ dT(dA, A') dA \equiv \int_M \frac{\cos \theta \cos \vartheta}{|x-x'|^2} dA(x') dA(x) \]

3. Finite-finite beam (radiosity calc.)
   \[ T(A, A') \equiv \int_M \int_{M'} \frac{\cos \theta \cos \vartheta}{|x-x'|^2} dA(x') dA(x) \]

Probability of Ray Intersection

Probability of a ray hitting \( A' \) given that it hits \( A \)

\[ \Pr(A' \mid A) = \frac{T(A', A)}{T(A)} \]

\[ A' \]

\[ \int_M \int_{M'} \frac{\cos \theta \cos \vartheta}{|x-x'|^2} dA(x') dA(x) \]

\[ A \]

\[ T(A) = \pi A \]
Another Formulation

\[ T(A', A) = \int_{A'} \int_{A} \frac{\cos \theta \cos \theta'}{|x - x'|^2} \, dA(x') \, dA(x) \]
\[ = \pi \int_{A'} \int_{A} G(x, x') \, dA(x') \, dA(x) \]
\[ G(x, x') = \frac{\cos \theta \cos \theta'}{\pi |x - x'|^2} V(x, x') \]
\[ V(x, x') = \begin{cases} 
0 & \text{not visible} \\
1 & \text{visible} 
\end{cases} \]

Form Factor

Probability of a ray hitting \( A' \) given it hits \( A \)

\[ \Pr(A' \mid A) \, T(A) = \Pr(A \mid A') \, T(A) = T(A', A) \]

Form factor definition

\[ F(A', A) = \Pr(A' \mid A) \]
\[ F(A, A') = \Pr(A \mid A') \]

Form factor reciprocity

\[ F(A', A) A = F(A, A') A' \]

CS348B Lecture 5  Pat Hanrahan, 2005
Radiosity

Power transfer from a constant radiance source

\[ \Phi(A, A') = L(A')T(A, A') \]
\[ = L(A')T(A') \frac{T(A, A')}{T(A')} \]
\[ = \Phi(A')F(A, A') \]

\[ E(A) = B(A')F(A', A) \]

Set up system of equations representing power transfers between objects

Form Factors and Throughput

Form factors represent the probability of ray leaving a surface intersecting another surface

- Only a function of surface geometry

Differential form factor

- Irradiance calculations

Form factors

- Radiosity calculations (energy balance)