Participating Media & Vol. Scattering

Applications
- Clouds, smoke, water, ...
- Subsurface scattering: paint, skin, ...
- Scientific/medical visualization: CT, MRI, ...

Topics
- Absorption and emission
- Scattering and phase functions
- Volume rendering equation
- Homogeneous media
- Ray tracing volumes

Absorption

\[ dL(x, \omega) = -\sigma_a(x)L(x, \omega) \, ds \]

Absorption cross-section: \( \sigma_a(x) \)

Probability of being absorbed per unit length
Transmittance

\[ dL(x, \omega) = -\sigma_a(x) L(x, \omega) \, ds \]
\[ \frac{dL(x, \omega)}{L(x, \omega)} = -\sigma_a(x) \, ds \]
\[ \ln L(x + s \omega, \omega) = - \int_0^s \sigma_a(x + s' \omega) \, ds' = -\tau(s) \]

Optical distance or depth

\[ \tau(s) = \int_0^s \sigma_a(x + s' \omega) \, ds' \]

Homogenous media: constant \( \sigma_a \)

\[ \sigma_a \rightarrow \tau(s) = \sigma_a s \]

Transmittance and Opacity

\[ dL(x, \omega) = -\sigma_a(x) L(x, \omega) \, ds \]
\[ \frac{dL(x, \omega)}{L(x, \omega)} = -\sigma_a(x) \, ds \]
\[ \ln L(x + s \omega, \omega) = - \int_0^s \sigma_a(x + s' \omega) \, ds' = -\tau(s) \]
\[ L(x + s \omega, \omega) = e^{-\tau(s)} L(x, \omega) = T(s) L(x, \omega) \]

Transmittance

\[ T(s) = e^{-\tau(s)} \]

Opacity

\[ \alpha(s) = 1 - T(s) \]
**Out-Scatter**

\[ L(x, \omega) \xrightarrow{\sigma_i(x)} L + dL \]

\[ dL(x, \omega) = -\sigma_s(x)L(x, \omega)\, ds \]

**Scattering cross-section:** \( \sigma_s \)

**Probability of being scattered per unit length**

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**Extinction**

\[ L(x, \omega) \xrightarrow{\sigma_i(x)} L + dL \]

\[ dL(x, \omega) = -\sigma_s(x)L(x, \omega)\, ds \]

**Total cross-section**  
\[ \sigma_t = \sigma_a + \sigma_s \]

**Albedo**  
\[ W = \frac{\sigma_s}{\sigma_t} = \frac{\sigma_s}{\sigma_a + \sigma_s} \]

**Attenuation due to both absorption and scattering**  
\[ \tau(s) = \int_0^s \sigma_t(x + s' \omega)\, ds' \]
Black Clouds

From Greenler, Rainbows, halos and glories

In-Scatter

\[ L(x, \omega) \xrightarrow{\sigma_s(x)} L + dL \]

\[ S(x, \omega) = \sigma_s(x) \int_{S^2} p(\omega' \rightarrow \omega) L(x, \omega') d\omega' \]

Phase function \[ p(\omega' \rightarrow \omega) \]

Reciprocity \[ p(\omega \rightarrow \omega') = p(\omega' \rightarrow \omega) \]

Energy conserving \[ \int_{S^2} p(\omega' \rightarrow \omega) d\omega' = 1 \]
Phase Functions

Phase angle \( \cos \theta = \omega \cdot \omega' \)

Phase functions
(from the phase of the moon)

1. Isotropic
   - simple
   \[ p(\cos \theta) = \frac{1}{4\pi} \]
2. Rayleigh
   - molecules
   \[ p(\cos \theta) = \frac{3}{4} \frac{1 + \cos^2 \theta}{\lambda^4} \]
3. Mie scattering
   - small spheres
   ... Huge literature ...

Blue Sky = Red Sunset

From Greenler, Rainbows, halos and glories
Coronas and Halos

Moon Corona  Sun Halos

From Greenler, Rainbows, halos and glories

Henyey-Greenstein Phase Function

Empirical phase function

\[
p(\cos \theta) = \frac{1}{4\pi} \frac{1 - g^2}{\left(1 + g^2 - 2g \cos \theta\right)^{3/2}}
\]

\[
2\pi \int_0^{\pi} p(\cos \theta) \cos \theta \, d\theta = g
\]

\( g \): average phase angle

\( g = -0.3 \)

\( g = 0.6 \)
The Volume Rendering Equation

Integro-differential equation
\[ \frac{\partial L(x, \omega)}{\partial s} = -\sigma_t(x)L(x, \omega) + S(x, \omega) \]

Integro-integral equation
\[ L(x, \omega) = \int_0^{\infty} S(x + s' \omega) ds' \]

Attenuation: Absorption and scattering
Source: Scatter (+ emission)

Simple Atmosphere Model

Assumptions
- Homogenous media
- Constant source term (airlight)

\[ \frac{\partial L(s)}{\partial s} = -\sigma_t L(s) + S \]
\[ L(s) = \left( 1 - e^{-\sigma_c s} \right) S + e^{-\sigma_c} C \]

Fog
Haze
The Sky

From Greenler, Rainbows, halos and glories

Atmospheric Perspective

From Greenler, Rainbows, halos and glories
Atmospheric Perspective

Aerial Perspective: loss of contrast and change in color

From Musgrave

Semi-Infinite Homogenous Media

Reduced Intensity

\[ L(z, \omega_i) = e^{-\tau(z, \omega_i)} L(0, \omega_i) \]

Effective source term

\[ S(z, \omega_o) = \sigma_s p(\omega_i \rightarrow \omega_o) e^{-\tau(z, \omega_o)} L(0, \omega_i) \]

Volume rendering equation

\[ \cos \theta_o \frac{\partial L(z, \omega_o)}{\partial z} = -\sigma_s L(z, \omega_o) + S(z, \omega_o) \]

Integrating over depths

\[ \cos \theta_o L(\omega_o) = \int_0^\infty e^{-\sigma z \cos \theta} \sigma_s p(\omega_i, \omega_o) e^{-\sigma z \cos \theta} L(\omega_i) \, dz \]
**Semi-Infinite Homogenous Media**

**Integrating over depths**

\[
\cos \theta_i L(\omega_o) = \int_0^\infty e^{-\sigma_z \cos \theta_i} \sigma_i p(\omega_i, \omega_o) e^{-\sigma_z \cos \theta_o} L(\omega_i) \, dz
\]

\[
= \sigma_i p(\omega_i, \omega_o) L(\omega_i) \int_0^\infty e^{-\sigma_i z} \left( \frac{1}{\cos \theta_i} + \frac{1}{\cos \theta_o} \right) \, dz
\]

\[
= \sigma_i p(\omega_i, \omega_o) L(\omega_i) \frac{1}{\cos \theta_i + \cos \theta_o}
\]

\[
= W \, p(\omega_i, \omega_o) \frac{\cos \theta_i \cos \theta_o}{\cos \theta_i + \cos \theta_o}
\]

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**Semi-Infinite Homogenous Media**

**BRDF**

\[
f_r(\omega_i, \omega_o) = \frac{dL}{dE} = \frac{L(\omega_i, \omega_o)}{L(\omega_i) \cos \theta_i}
\]

\[
= W \, p(\omega_i, \omega_o) \frac{1}{\cos \theta_i + \cos \theta_o}
\]

**Seeliger’s Law or The Law of Diffuse Reflection**
Subsurface Scattering

Skin

Volume Representations

3D arrays (uniform rectangular)
- CT data

3D meshes
- CFD, mechanical simulation

Simple shapes with solid texture
- Ellipsoidal clouds with sum-of-sines densities
- Hypertexture
Scalar Volumes

Interpolation \[ v(s_i) = \text{trilinear}(v,i,j,k,x(s_i)) \]

Map scalars to optical properties \( \sigma_r(v), \sigma_a(v) \)

Scalar Volumes

Scatter

\[ S(x(s), \omega) = \sigma_z(s) \ p(\omega, \omega(x(s), x_L)) \ L_z(x_L, \omega(x_L, x(s))) \]

\( (i, j, k) \)

Voxel
Ray Marching

Primary ray

\[ T = 1 \]
\[ L = 0 \]

for \( s = 0; s < 1; s+ = ds \)

\[ S = \sigma_s(s) p(\omega, \omega(x(s)), x_L) L_s(x_L, \omega(x_L, x(s))) \]
\[ L = L + TS\Delta s \]
\[ T = T \left[ 1 - \sigma_s(x(s)) \right] \Delta s \]

Ray Marching

Shadow ray

\[ T = 1 \]

for \( t = 0; t < 1; t+ = dt \)

\[ T = T \left[ 1 - \sigma_t(x(t)) \right] \Delta t \]
\[ S(x(s)) = \sigma_t(s) p(\omega, \omega(x(s)), x_L) TL_s(x_L, \omega(x_L, x(s))) \]
Beams of Light

From Greenler, Rainbows, halos and glories

From Minneart, Color and light in the open air

Color and Opacity Volumes

M. Levoy, Ray tracing volume densities

\[ C(i,j,k) \Rightarrow (R,G,B) \]

\[ A(i,j,k) \]

\[ c(i,j,k) = \]

\[ (C(i,j,k)^* A(i,j,k), A(i,j,k)) \]

\[ c(x(s_j)) = \text{trilinear}(c,i,j,k,x(s_j)) \]
Ray Marching

\[ C = (0, 0, 0, 0) \]
\[ \text{for} (s = 0; s < 1; s+ = ds) \]
\[ C = C + (1 - \alpha(C))c(s) \]

Volume Rendering Examples

From Karl Heinz Hoehne

From Marc Levoy

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