## Camera Simulation

<table>
<thead>
<tr>
<th>Effect</th>
<th>Cause</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field of view</td>
<td>Film size, stops and pupils</td>
</tr>
<tr>
<td>Depth of field</td>
<td>Aperture, focal length</td>
</tr>
<tr>
<td>Motion blur</td>
<td>Shutter</td>
</tr>
<tr>
<td>Exposure</td>
<td>Film speed, aperture, shutter</td>
</tr>
</tbody>
</table>

### References
- Photography, B. London and J. Upton
- Optics in Photography, R. Kingslake
- The Camera, The Negative, The Print, A. Adams

## Topics
- Lenses and field of view
- Depth of focus and depth of field
- Exposure
Lenses

Refraction

Snell’s Law

\[ n' \sin I' = n \sin I \]
Paraxial Approximation

\[ \sin U \approx u \]
\[ \tan U \approx u \]

Rays deviate only slightly from the axis

Incident Ray

Angles: ccw is positive; cw is negative

\[ I = U + (-\phi) \]

The sum of the interior angles is equal to the exterior angle.
Refracted Ray

\[ I' = U' - \phi \]

Derivation

Paraxial approximation

\[ n' \sin I' = n \sin I \Rightarrow n' i' = n i \]

\[ I = U - \phi \Rightarrow i = u - \phi \]

\[ I' = U' - \phi \Rightarrow i' = u' - \phi \]

\[ n'(u' - \phi) = n(u - \phi) \]
Ray Coordinates

\[ u = \frac{h}{-z} \]

\[ a \approx \sin A \approx \tan A \]
Ray Coordinates

\[ u = \frac{h}{-z} \quad -u' = \frac{h}{z'} \]

Ray Coordinates

\[ u = \frac{h}{-z} \quad -u' = \frac{h}{z'} \quad -\phi = \frac{h}{R} \]
**Gauss’ Formula**

Paraxial approximation to Snell’s Law

\[ n'(u' - \phi) = n(u - \phi) \]

Ray coordinates

\[ u' = -\frac{h}{z'} \quad \phi = -\frac{h}{R} \quad u = -\frac{h}{z} \]

Thin lens equation

\[ n'(\frac{h}{z'} - \frac{h}{R}) = n(\frac{h}{z} - \frac{h}{R}) \]

\[ \frac{n'}{z'} = \frac{n}{z} + \frac{(n' - n)}{R} \quad \text{Holds for any height, any ray!} \]

---

**Vergence**

Diverging  
\[ V < 0 \]

Converging  
\[ V > 0 \]

\[ V = 0 \]

Vergence

\[ V \equiv \frac{n}{r} \approx \frac{n}{z} \quad \left[ \frac{1}{m} = \text{diopters} \right] \]

Thin lens equation

\[ V' = V + P \]

Surface Power equation

\[ P \equiv (n' - n) \frac{1}{R} \]
**Lens-makers Formula**

**Refractive Power**

\[ P = (n' - n) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) = \frac{1}{f} \]

Converging Diverging

**Conjugate Points**

\[ \frac{1}{z'} = \frac{1}{z} + \frac{1}{f} \]

To focus: move lens relative to backplane
Horizontal rays converge on focal point in the focal plane
Gauss’ Ray Tracing Construction

Parallel Ray

Focal Ray

Chief Ray

Object

Image

Ray Tracing: Finite Aperture

Focal Plane

Aperture Plane

Back Plane
### Real Lens

Cutaway section of a Vivitar Series 1 90mm f/2.5 lens
Cover photo, Kingslake, *Optics in Photography*

### Double Gauss

#### Data from W. Smith, *Modern Lens Design*, p 312

<table>
<thead>
<tr>
<th>Radius (mm)</th>
<th>Thick (mm)</th>
<th>$n_d$</th>
<th>V-no</th>
<th>Aperture</th>
</tr>
</thead>
<tbody>
<tr>
<td>58.950</td>
<td>7.520</td>
<td>1.670</td>
<td>47.1</td>
<td>50.4</td>
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<tr>
<td>169.660</td>
<td>0.240</td>
<td></td>
<td></td>
<td>50.4</td>
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<tr>
<td>38.550</td>
<td>8.050</td>
<td>1.670</td>
<td>47.1</td>
<td>46.0</td>
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<tr>
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<td>6.550</td>
<td>1.699</td>
<td>30.1</td>
<td>46.0</td>
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<tr>
<td>25.500</td>
<td>11.410</td>
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<td></td>
<td>36.0</td>
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<tr>
<td>9.000</td>
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<td></td>
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<td>34.2</td>
</tr>
<tr>
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<td>34.0</td>
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<tr>
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<tr>
<td>874.130</td>
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<td>1.717</td>
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<td>40.0</td>
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<tr>
<td>-79.460</td>
<td>72.228</td>
<td></td>
<td></td>
<td>40.0</td>
</tr>
</tbody>
</table>
Ray Tracing Through Lenses

200 mm telephoto  35 mm wide-angle
50 mm double-gauss  16 mm fisheye

From Kolb, Mitchell and Hanrahan (1995)

Fact 1: Thick Lenses

Equivalent Lens

Refraction occurs at the principal planes
Fact 2: Perspective Transformation

Thin lens equation

\[
\frac{1}{z'} = \frac{1}{z} + \frac{1}{f} \Rightarrow z' = \frac{fz}{z + f} \\
\Rightarrow x' = \frac{fx}{z + f} \\
\Rightarrow y' = \frac{fy}{z + f}
\]

Represent transformation as a 4x4 matrix

Fact 3: Paraxial Ray Tracing

Characterize ray by \((y, v)\) where \(v = nu\)

<table>
<thead>
<tr>
<th>Refraction</th>
<th>Transfer</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y' = y)</td>
<td>(y' = y + (d/n)v)</td>
</tr>
<tr>
<td>(v' = v + P)</td>
<td>(v' = v)</td>
</tr>
</tbody>
</table>

\[R = \begin{bmatrix} 1 & 0 \\ P & 0 \end{bmatrix} \quad T = \begin{bmatrix} 1 & d/n \\ 0 & 1 \end{bmatrix}\]
Field of View

From London and Upton
Field of View

Field of view

\[
\tan \frac{\text{fov}}{2} = \frac{\text{filmsize}}{f}
\]

Types of lenses

- Normal 26°
  Film diagonal ~ focal length
- Wide-angle 75-90°
- Narrow-angle 10°

Redrawn from Kingslake, *Optics in Photography*
Depth of Field

From London and Upton
**Circle of Confusion**

Circle of confusion proportional to the size of the aperture

\[
\frac{c}{a} = \frac{d'}{z'} = \frac{s'-z'}{z'}
\]

**Depth of Focus [Image Space]**

Depth of focus =

Equal circles of confusion

Two planes: near and far

\[
\frac{c}{a} = \frac{d'_f}{z'_f} = \frac{s'-z'_f}{z'_f}
\]

\[
\frac{c}{a} = \frac{d'_n}{z'_n} = \frac{z'_n-s'}{z'_n}
\]
\[ \text{Depth of focus} \equiv \text{Equal circles of confusion} \]

\[ \frac{1}{z_f'} = \frac{1}{s'} \left( 1 + \frac{c}{a} \right) \quad \frac{1}{z_n'} = \frac{1}{s'} \left( 1 - \frac{c}{a} \right) \]

\[ \frac{1}{z_f'} + \frac{1}{z_n'} = 2 \frac{1}{s'} \]

\[ \frac{1}{z_f'} - \frac{1}{z_n'} = 2c \frac{1}{a} \frac{1}{s'} \]
Depth of Field [Object Space]

Depth of field ≡
Equal circles of confusion

\[
\frac{1}{s'} = \frac{1}{s} + \frac{1}{f} \quad \frac{1}{z'_n} = \frac{1}{z_n} + \frac{1}{f} \quad \frac{1}{z'_f} = \frac{1}{z_f} + \frac{1}{f}
\]

\[
\frac{1}{z_n} + \frac{1}{z_f} = 2 \frac{1}{s}
\]

\[
\frac{1}{z_n} - \frac{1}{z_f} = 2c \left( \frac{1}{a} - \frac{1}{s} \right) \approx \frac{2c}{a} \frac{1}{f}
\]

Hyperfocal Distance

\[
\frac{1}{z_n} + \frac{1}{z_f} = 2 \frac{1}{s} \quad N \equiv \frac{a}{f}
\]

\[
\frac{1}{z_n} - \frac{1}{z_f} = 2c \frac{1}{a} - \frac{2cN}{f^2} \equiv 2 \frac{1}{H}
\]

When

\[s \to H \Rightarrow z_n = \frac{H}{2}, z_f = \infty\]

\[H \text{ is the hyperfocal distance}\]
Depth of Field Scale

Factors Affecting DOF

\[ \frac{1}{H} = \frac{cN}{f^2} \]
Resolving Power

- Diffraction limit
  \[ c = 1.22 \frac{f}{\lambda} \times \frac{\lambda}{d} \quad [= 1.22 \times 64 \times .500 \mu m = 0.040 \, mm] \]

- 35mm film (Leica standard)
  \[ c = 0.025 \, mm \]

- CCD/CMOS pixel aperture
  \[ c = 0.0116 \, mm \text{ (Nikon D1)} \]
**Image Irradiance**

\[ E = L \pi \sin^2 \theta = L \frac{\pi}{4} \left( \frac{a}{f} \right)^2 \]

**Relative Aperture or F-Stop**

\[ a = \frac{f}{N} \]

**F-Number and exposure:** 
\[ E = L \frac{\pi}{4} \frac{1}{N^2} \]

**Fstops:** 1.4 2 2.8 4.0 5.6 8 11 16 22 32 45 64

1 stop doubles exposure
Camera Exposure

**Exposure** \( H = E \times T \)

Exposure overdetermined

- **Aperture**: f-stop - 1 stop doubles \( H \)
  - Decreases depth of field

- **Shutter**: Doubling the open time doubles \( H \)
  - Increases motion blur

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Aperture vs Shutter

- \( f/16 \) 1/8s
- \( f/4 \) 1/125s
- \( f/2 \) 1/500s

*From London and Upton*
High Dynamic Range

Sixteen photographs of the Stanford Memorial Church taken at 1-stop increments from 30s to 1/1000s.
From Debevec and Malik, High dynamic range photographs.

Simulated Photograph

Adaptive histogram
With glare, contrast, blur
Camera Simulation

\[ R = \iiint_{A \Omega t \lambda} P(x', \lambda) S(x', \omega', t) L(T(x', \omega', \lambda), t, \lambda) d\Omega d\omega' dt d\lambda \]

**Sensor response**  \( P(x', \lambda) \)

**Lens**  \( (x, \omega) = T(x', \omega', \lambda) \)

**Shutter**  \( S(x', \omega', t) \)

**Scene radiance**  \( L(x, \omega, t, \lambda) \)