Reflection Models I

Today
- Types of reflection models
- The BRDF and reflectance
- The reflection equation
- Ideal reflection and refraction
- Fresnel effect
- Ideal diffuse

Next lecture
- Glossy and specular reflection models
- Rough surfaces and microfacets
- Self-shadowing
- Anisotropic reflection models

Reflection Models

Definition: Reflection is the process by which light incident on a surface interacts with the surface such that it leaves on the incident side without change in frequency.

Properties
- Spectra and Color [Moon Spectra]
- Polarization
- Directional distribution

Theories
- Phenomenological
- Physical
Types of Reflection Functions

Ideal Specular
- Reflection Law
- Mirror

Ideal Diffuse
- Lambert’s Law
- Matte

Specular
- Glossy
- Directional diffuse

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Materials

Plastic  Metal  Matte

From Apodaca and Gritz, Advanced RenderMan

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The Reflection Equation

\[ L_r(x, \omega_r) = \int f_r(x, \omega_i \rightarrow \omega_r) L_i(x, \omega_i) \cos \theta_i \, d\omega_i \]

The BRDF

Bidirectional Reflectance-Distribution Function

\[ f_r(\omega_i \rightarrow \omega_r) \equiv \frac{dL_i(\omega_i \rightarrow \omega_r)}{dE_i} \left[ \frac{1}{sr} \right] \]
The BSSRDF

Bidirectional Surface Scattering Reflectance-Distribution Function

\[ dL_i(x, \omega_i) \rightarrow dL_i(x, \omega_i) \equiv \frac{dL_i(x, \omega_i \rightarrow x, \omega_i)}{d\Phi_i} \]

Translucency

Gonioreflectometer

4 degree-of-freedom gantry
Properties of BRDF’s

1. Linear

2. Reciprocity principle \( f_r(\omega_r \rightarrow \omega_i) = f_r(\omega_i \rightarrow \omega_r) \)

3. Isotropic vs. anisotropic

   \( f_r(\theta_i, \varphi_i; \theta_r, \varphi_r) = f_r(\theta_i, \theta_r, \varphi_r - \varphi_i) \)

   Reciprocity and isotropy

   \( f_r(\theta_i, \theta_r, \varphi_r - \varphi_i) = f_r(\theta_r, \theta_i, \varphi_i - \varphi_r) = f_r(\theta_i, \theta_r, |\varphi_r - \varphi_i|) \)

4. Energy conservation
Energy Conservation

\[
\frac{d\Phi_r}{d\Phi_i} = \frac{\int_L L_i(\omega_r) \cos \theta_r \, d\omega_r}{\int_L L_i(\omega_i) \cos \theta_i \, d\omega_i} = \frac{\int f_r(\omega_i \to \omega_r)L_i(\omega_i) \cos \theta_i \, d\omega_i \cos \theta_r \, d\omega_r}{\int L_i(\omega_i) \cos \theta_i \, d\omega_i} \leq 1
\]

The Reflectance

Definition: Reflectance is ratio of reflected to incident power

\[
\rho(\Omega_i \to \Omega_r) \equiv \frac{\int f_r(\omega_i \to \omega_r) \cos \theta_i \, d\omega_i \cos \theta_r \, d\omega_r}{\int \cos \theta_i \, d\omega_i} \leq \frac{\int f_r(\omega_i \to \omega_r)L_i(\omega_i) \cos \theta_i \, d\omega_i \cos \theta_r \, d\omega_r}{\int L_i(\omega_i) \cos \theta_i \, d\omega_i}
\]

Conservation of energy: \(0 < \rho < 1\)

3 by 3 set of possibilities: \(\{d\omega_i, \Omega_i, H_i^2\} \times \{d\omega_i, \Omega_i, H_r^2\}\)

Units: \(\rho\) [dimensionless], \(f_r\) [1/steradians]
**Law of Reflection**

\[ \hat{R} + (-\hat{I}) = 2 \cos \theta \hat{N} = -2(\hat{I} \cdot \hat{N})\hat{N} \]

\[ \hat{R} = \hat{I} - 2(\hat{I} \cdot \hat{N})\hat{N} \]

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**Ideal Reflection (Mirror)**

\[ f_{r,m}(\theta_i, \varphi_i; \theta_r, \varphi_r) = \frac{\delta(\cos \theta_i - \cos \theta_r)}{\cos \theta_i} \delta(\varphi_i - \varphi_r \pm \pi) \]

\[ L_{r,m}(\theta_r, \varphi_r) = \int f_{r,m}(\theta_i, \varphi_i; \theta_r, \varphi_r) L_i(\theta_i, \varphi_i) \cos \theta_i d\cos \theta_i d\varphi_i \]

\[ = \int \frac{\delta(\cos \theta_i - \cos \theta_r)}{\cos \theta_i} \delta(\varphi_i - \varphi_r \pm \pi)L_i(\theta_i, \varphi_i) \cos \theta_i d\cos \theta_i d\varphi_i \]

\[ = L_i(\theta_r, \varphi_r \pm \pi) \]
**Snell’s Law**

\[ n_i \sin \theta_i = n_\parallel \sin \theta_i \]

\[ n_i \hat{N} \times \hat{I} = n_\parallel \hat{N} \times \hat{T} \]

**Law of Refraction**

\[ \hat{N} \times \hat{T} = \mu \hat{N} \times \hat{I} \]

\[ \hat{N} \times (\hat{T} - \mu \hat{I}) = 0 \]

\[ \hat{T} = \mu \hat{I} + \gamma \hat{N} \]

\[ \gamma = -\mu \cdot \hat{I} \cdot \hat{N} \pm \left( 1 - \mu^2 \left( 1 - (\hat{I} \cdot \hat{N})^2 \right) \right)^{1/2} \]

Total internal reflection:

\[ 1 - \mu^2 \left( 1 - (\hat{I} \cdot \hat{N})^2 \right) < 0 \]
**Optical Manhole**

Total internal reflection

\[ n_{\text{w}} = \frac{4}{3} \]

From Livingston and Lynch

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**Fresnel Reflectance**

**Metal (Aluminum)**

Gold \( F(0) = 0.82 \)
Silver \( F(0) = 0.95 \)

**Dielectric (N=1.5)**

Glass \( \text{n}=1.5 \) \( F(0)=0.04 \)
Diamond \( \text{n}=2.4 \) \( F(0)=0.15 \)

Schlick Approximation

\[ F(\theta) = F(0) + (1 - F(0))(1 - \cos \theta)^5 \]
Experiment

Reflections from a shiny floor

From Lafortune, Foo, Torrance, Greenberg, SIGGRAPH 97

Cook-Torrance Model for Metals

Reflectance of Copper as a function of wavelength and angle of incidence

Light spectra

Measured Reflectance

Approximated Reflectance

Cook-Torrance approximation

\[
R = R(0) + R(\pi/2) \left( \frac{F(\theta) - F(0)}{F(\pi/2) - F(0)} \right)
\]

Copper spectra
Ideal Diffuse Reflection

Assume light is equally likely to be reflected in any output direction (independent of input direction).

\[ L_{r,d}(\omega_r) = \int f_{r,d} L_i(\omega_i) \cos \theta_i \, d\omega_i \]
\[ = f_{r,d} \int L_i(\omega_i) \cos \theta_i \, d\omega_i \]
\[ = f_{r,d} E \]

\[ f_{r,d} = c \]

\[ M = \int L_r(\omega_r) \cos \theta_r \, d\omega_r = L_r \int \cos \theta_r \, d\omega_r = \pi L_r \]

\[ \rho_d = \frac{M}{E} = \frac{\pi L_r}{E} = \pi \frac{f_{r,d} E}{\pi} = \pi f_{r,d} \quad \Rightarrow \quad f_{r,d} = \frac{\rho_d}{\pi} \]

Lambert’s Cosine Law \[ M = \rho_d E = \rho_d E \cos \theta_i \]

“Diffuse” Reflection

Theoretical

- Bouguer - Special micro-facet distribution
- Seeliger - Subsurface reflection
- Multiple surface or subsurface reflections

Experimental

- Pressed magnesium oxide powder
- Almost never valid at high angles of incidence

Paint manufacturers attempt to create ideal diffuse
Phong Model

\[
\begin{align*}
(\hat{E} \cdot R(\hat{L}))^\gamma & \quad (\hat{L} \cdot R(\hat{E}))^\gamma \\
\text{Reciprocity:} & \quad (\hat{E} \cdot R(\hat{L}))^\gamma = (\hat{L} \cdot R(\hat{E}))^\gamma
\end{align*}
\]

Distributed light source!

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Phong Model

Mirror

Diffuse
Properties of the Phong Model

Energy normalize Phong Model

\[
\rho(H^2 \rightarrow \omega_r) = \int_{H^2(S)} \left( \mathbf{L} \cdot \mathbf{R}(\hat{\mathbf{E}}) \right)^\tau \cos \theta_i d\omega_i
\]
\[
\leq \int_{H^2(R)} \left( \mathbf{L} \cdot \mathbf{R}(\hat{\mathbf{E}}) \right)^\tau d\omega_{ir}
\]
\[
\leq \int_{H^2} \cos^\tau \theta d\omega = \frac{2\pi}{s + 1}
\]