Participating Media & Vol. Scattering

Applications

- Clouds, smoke, water, ...
- Subsurface scattering: paint, skin, ...
- Scientific/medical visualization: CT, MRI, ...

Topics

- Absorption and emission
- Scattering and phase functions
- Volume rendering equation
- Homogeneous media
- Ray tracing volumes

Absorption

\[
\begin{align*}
L(x, \omega) & \xrightarrow{\sigma_a(x)} L + dL \\
\text{Absorption cross-section: } & \sigma_a(x) \\
\text{Probability of being absorbed per unit length: } & \end{align*}
\]

\[
dL(x, \omega) = -\sigma_a(x)L(x, \omega)ds
\]
Transmittance

\[ dL(x, \omega) = -\sigma_a(x)L(x, \omega) \, ds \]
\[ \frac{dL(x, \omega)}{L(x, \omega)} = -\sigma_a(x) \, ds \]
\[ \ln L(x + s \omega, \omega) = \int_0^s \sigma_a(x + s' \omega) \, ds' = -\tau(s) \]

Optical distance or depth

\[ \tau(s) = \int_0^s \sigma_a(x + s' \omega) \, ds' \]

Homogenous media: constant \( \sigma_a \)

\[ \sigma_a \rightarrow \tau(s) = \sigma_a s \]

Transmittance and Opacity

\[ dL(x, \omega) = -\sigma_a(x)L(x, \omega) \, ds \]
\[ \frac{dL(x, \omega)}{L(x, \omega)} = -\sigma_a(x) \, ds \]
\[ \ln L(x + s \omega, \omega) = \int_0^s \sigma_a(x + s' \omega) \, ds' = -\tau(s) \]
\[ L(x + s \omega, \omega) = e^{-\tau(s)}L(x, \omega) = T(s)L(x, \omega) \]

Transmittance

\[ T(s) = e^{-\tau(s)} \]

Opacity

\[ \alpha(s) = 1 - T(s) \]
Out-Scatter

\[ dL(x, \omega) = -\sigma_s(x)L(x, \omega)\, ds \]

**Scattering cross-section:** \( \sigma_s \)

*Probability of being absorbed per unit length*

Extinction

\[ dL(x, \omega) = -\sigma_a(x)L(x, \omega)\, ds \]

**Total cross-section**

\[ \sigma_t = \sigma_a + \sigma_s \]

**Albedo**

\[ W = \frac{\sigma_s}{\sigma_t} = \frac{-\sigma_s}{\sigma_a + \sigma_s} \]

**Attenuation due to both absorption and scattering**

\[ \tau(s) = \int_0^s \sigma_t(x + s' \omega)\, ds' \]
Black Clouds

From Greenler, Rainbows, halos and glories

In-Scatter

\[ L(x, \omega) \rightarrow \sigma_s(x) \rightarrow L + dL \]

\[ S(x, \omega) = \sigma_s(x) \int_{S^2} p(\omega' \rightarrow \omega) L(x, \omega') d\omega' \]

Phase function \( p(\omega' \rightarrow \omega) \)

Reciprocal \( p(\omega \rightarrow \omega') = p(\omega' \rightarrow \omega) \)

Normalized \( \int_{S^2} p(\omega' \rightarrow \omega) d\omega' = 1 \)
# Phase Functions

**Phase angle** \( \cos \theta = \omega \cdot \omega' \)

Phase functions
(from the phase of the moon)

1. **Isotropic**
   - simple
   \[ p(\cos \theta) = \frac{1}{4\pi} \]

2. **Rayleigh**
   - molecules
   \[ p(\cos \theta) = \frac{3}{4} \frac{1 + \cos^2 \theta}{\lambda^4} \]

3. **Mie scattering**
   - small spheres
   ... Huge literature ...

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# Blue Sky = Red Sunset

From Greenler, Rainbows, halos and glories
**Coronas and Halos**

**Moon Corona**

**Sun Halos**

*From Greenler, Rainbows, halos and glories*

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**Henyey-Greenstein Phase Function**

**Empirical phase function**

\[
p(\cos \theta) = \frac{1}{4\pi} \frac{1 - g^2}{(1 + g^2 - 2g \cos \theta)^{3/2}}
\]

\[
g = \int_0^{\pi} p(\cos \theta) \cos \theta \, d\theta = 0.3
\]

\[
g = 0.6
\]

\[
\text{g: average phase angle}
\]
The Volume Rendering Equation

Integro-differential equation

\[ \frac{\partial L(x, \omega)}{\partial s} = -\sigma_{s}(x)L(x, \omega) + S(x, \omega) \]

Integro-integral equation

\[ L(x, \omega) = \int_{0}^{\infty} e^{-\int_{0}^{s} \sigma_{s}(x + s'\omega) \, ds'} S(x + s'\omega) \, ds' \]

Attenuation: Absorption and scattering

Source: Scatter (+ emission)

Simple Atmosphere Model

Assumptions

- Homogenous media
- Constant source term (airlight)

\[ \frac{\partial L(s)}{\partial s} = -\sigma_{s}L(s) + S \]

\[ L(s) = \left(1 - e^{-\sigma_{s}s}\right)S + e^{-\sigma_{s}}C \]

Fog

Haze
The Sky

From Greenler, Rainbows, halos and glories

Atmospheric Perspective

From Greenler, Rainbows, halos and glories
Atmospheric Perspective

Aerial Perspective: loss of contrast and change in color

From Musgrave

Semi-Infinite Homogenous Media

Reduced Intensity

\[ L(z, \omega_i) = e^{-\tau(z, \omega_o)} L(0, \omega_i) \]

Effective source term

\[ S(z, \omega_o) = \sigma_s p(\omega_i \rightarrow \omega_o) e^{-\tau(z, \omega_o)} L(0, \omega_i) \]

Volume rendering equation

\[ \cos \theta_o \frac{\partial L(z, \omega_o)}{\partial z} = -\sigma_s L(z, \omega_o) + S(z, \omega_o) \]

Integrating over depths

\[ \cos \theta_o L(\omega_o) = \int_0^\infty e^{-\tau(z, \omega_o)} \sigma_s p(\omega_i, \omega_o) e^{-\sigma_s z \cos \theta} L(\omega_i) \, dz \]
### Semi-Infinite Homogenous Media

#### Integrating over depths

\[
\cos \theta_i L(\omega_i) = \int_0^\infty e^{-\frac{\sigma_i}{\cos \theta_i} \sigma_o} p(\omega_i, \omega_o) e^{-\frac{\sigma_o}{\cos \theta_o}} L(\omega_o) \, dz \\
= \sigma_i p(\omega_i, \omega_o) L(\omega_i) \int_0^\infty e^{-\frac{\sigma_i}{\cos \theta_i} \sigma_o} \frac{1}{\frac{1}{\cos \theta_i} + \frac{1}{\cos \theta_o}} \, dz \\
= W \ p(\omega_i, \omega_o) L(\omega_i) \frac{\cos \theta_i \cos \theta_o}{\cos \theta_i + \cos \theta_o}
\]

### BRDF

\[
f_r(\omega_i, \omega_o) = \frac{dL}{dE} = \frac{L(\omega_i, \omega_o)}{L(\omega_i) \cos \theta_i} \\
= W \ p(\omega_i, \omega_o) \frac{1}{\cos \theta_i + \cos \theta_o}
\]

**Seeliger’s Law or The Law of Diffuse Reflection**
Subsurface Scattering

Skin

Volume Representations

3D arrays (uniform rectangular)
- CT data

3D meshes
- CFD, mechanical simulation

Simple shapes with solid texture
- Ellipsoidal clouds with sum-of-sines densities
- Hypertexture
Scalar Volumes

Interpolation \( v(s_i) = \text{trilinear}(v, i, j, k, x(s_i)) \)

Map scalars to optical properties \( \sigma_s(v), \sigma_a(v) \)

Scalar Volumes

Scatter

\( S(x(s), \omega) = \sigma_s(s) p(\omega, \omega(x(s), x_L)) L_s(x_L, \omega(x_L, x(s))) \)
Ray Marching

Primary ray

\[ T = 1 \]
\[ L = 0 \]

for \( s = 0; s < 1; s^+ = ds \)
\[ S = \sigma_i(s) \rho(\omega, \omega(x(s), x_L)) L_s(x_L, \omega(x_L, x(s))) \]
\[ L = L + TS\Delta s \]
\[ T = T[1 - \sigma_i(x(s))]\Delta s \]

Ray Marching

Shadow ray

\[ T = 1 \]

for \( t = 0; t < 1; t^+ = dt \)
\[ T = T[1 - \sigma_i(x(t))]\Delta t \]
\[ S(x(s)) = \sigma_i(s) \rho(\omega, \omega(x(s), x_L)) TL_s(x_L, \omega(x_L, x(s))) \]
Beams of Light

From Greenler, Rainbows, halos and glories

From Minneart, Color and light in the open air

Color and Opacity Volumes

M. Levoy, Ray tracing volume densities

\[ C(i, j, k) \Rightarrow (R, G, B) \]
\[ A(i, j, k) \]
\[ c(i, j, k) = \]
\[ (C(i, j, k) * A(i, j, k), A(i, j, k)) \]
\[ c(x(s_j)) = \text{trilinear}(c, i, j, k, x(s_j)) \]
Ray Marching

\[ C = (0, 0, 0, 0) \]
\[ \text{for} (s = 0; s < 1; s+ = ds) \]
\[ C = C + (1 - \alpha(C))c(s) \]

Volume Rendering Examples

From Marc Levoy

From Karl Heinz Hoehne