Reflection Models I

Today
- Types of reflection models
- The BRDF and reflectance
- The reflection equation
- Ideal reflection and refraction
- Fresnel effect
- Ideal diffuse

Next lecture
- Glossy and specular reflection models
- Rough surfaces and microfacets
- Self-shadowing
- Anisotropic reflection models

Reflection Models

Definition: Reflection is the process by which light incident on a surface interacts with the surface such that it leaves on the incident side without change in frequency.

Properties
- Spectra and Color [Moon Spectra]
- Polarization
- Directional distribution

Theories
- Phenomenological
- Physical
Types of Reflection Functions

Ideal Specular
- Reflection Law
- Mirror

Ideal Diffuse
- Lambert’s Law
- Matte

Specular
- Glossy
- Directional diffuse

Materials

Plastic  Metal  Matte

From Apodaca and Gritz, Advanced RenderMan
The Reflection Equation

\[ L_r(x, \omega_r) = \int_{\omega_t} f_r(x, \omega_i \rightarrow \omega_r) L_i(x, \omega_i) \cos \theta_i \, d\omega_i \]

The BRDF

Bidirectional Reflectance-Distribution Function

\[ f_r(\omega_i \rightarrow \omega_r) = \frac{dL_s(\omega_i \rightarrow \omega_r)}{dE_i} \left[ \frac{1}{sr} \right] \]
The BSSRDF

Bidirectional Surface Scattering Reflectance-Distribution Function

\[ S(x_i, \omega_i \rightarrow x_r, \omega_r) \equiv \frac{dL_r(x_r, \omega_r \rightarrow x_i, \omega_i)}{d\Phi_i} \]

Translucency

Gonioreflectometer

4 degree-of-freedom gantry
Properties of BRDF’s

1. Linear

From Sillion, Arvo, Westin, Greenberg

2. Reciprocity principle $f_r(\omega_r \rightarrow \omega_i) = f_i(\omega_i \rightarrow \omega_r)$

3. Isotropic vs. anisotropic

$\hat{r}_i(\theta_i, \varphi_i; \theta_r, \varphi_r) = \hat{r}_i(\theta_i, \varphi_i - \varphi_r)$

Reciprocity and isotropy

$f_r(\theta_i, \theta_r, \varphi_i - \varphi_r) = f_r(\theta_i, \theta_r, \varphi_i) = f_r(\theta_i, \theta_r, |\varphi_i - \varphi_r|)$

4. Energy conservation
Energy Conservation

\[
d\Phi_i = \int_{\Omega_i} L_i(\omega_i) \cos \theta_i \, d\omega_i \int_{\Omega_i} L_i(\omega_i) \cos \theta_i \, d\omega_i
\]

\[
\int \int f_r(\omega_i \rightarrow \omega_r) L_i(\omega_i) \cos \theta_i \, d\omega_i \cos \theta_r \, d\omega_r
\]

\[
= \frac{\int \int \int f_r(\omega_i \rightarrow \omega_r) L_i(\omega_i) \cos \theta_i \, d\omega_i \cos \theta_r \, d\omega_r}{\int \Omega_i L_i(\omega_i) \cos \theta_i \, d\omega_i} \leq 1
\]

The Reflectance

Definition: Reflectance is ratio of reflected to incident power

\[
\rho(\Omega_i \rightarrow \Omega_r) \equiv \int \int f_r(\omega_i \rightarrow \omega_r) \cos \theta_i \, d\omega_i \cos \theta_r \, d\omega_r
\]

\[
\int \Omega_i \Omega_i \cos \theta_i \, d\omega_i
\]

\[
\int \int \int f_r(\omega_i \rightarrow \omega_r) L_i(\omega_i) \cos \theta_i \, d\omega_i \cos \theta_r \, d\omega_r
\]

\[
\leq \frac{\int \int \int f_r(\omega_i \rightarrow \omega_r) L_i(\omega_i) \cos \theta_i \, d\omega_i \cos \theta_r \, d\omega_r}{\int \Omega_i L_i(\omega_i) \cos \theta_i \, d\omega_i}
\]

Conservation of energy: \(0 < \rho < 1\)

3 by 3 set of possibilities: \(\{d\omega_i, \Omega_i, H_i^2\} \times \{d\omega_r, \Omega_r, H_r^2\}\)

Units: \(\rho \) [dimensionless], \(f_r \) [1/steradians]
Law of Reflection

\[ \hat{R} + (-\hat{I}) = 2 \cos \theta \hat{N} = -2(\hat{I} \cdot \hat{N})\hat{N} \]

\[ \hat{R} = \hat{I} - 2(\hat{I} \cdot \hat{N})\hat{N} \]

Ideal Reflection (Mirror)

\[ L_i(\theta_i, \varphi_i) \quad L_r(\theta_r, \varphi_r) \]

\[ r_{r,m}(\theta_r, \varphi_r; \theta_i, \varphi_i) = \frac{\delta(\cos \theta_i - \cos \theta_r)}{\cos \theta_i} \delta(\varphi_i - \varphi_r \pm \pi) \]

\[ L_{r,m}(\theta_r, \varphi_r) = \int r_{r,m}(\theta_r, \varphi_r; \theta_i, \varphi_i) L_i(\theta_i, \varphi_i) \cos \theta_i \, d \cos \theta_i \, d \varphi_i \]

\[ = \int \frac{\delta(\cos \theta_i - \cos \theta_r)}{\cos \theta_i} \delta(\varphi_i - \varphi_r \pm \pi) L_i(\theta_i, \varphi_i) \cos \theta_i \, d \cos \theta_i \, d \varphi_i \]

\[ = L_i(\theta_r, \varphi_r \pm \pi) \]
Snell’s Law

\[ \sin \theta_i \sin \theta_t = n_i \sin \theta_i \]

\[ n_i \hat{N} \times \hat{I} = n_i \hat{N} \times \hat{T} \]

Law of Refraction

\[ \hat{N} \times \hat{T} = \mu \hat{N} \times \hat{I} \]

\[ \hat{N} \times (\hat{T} - \mu \hat{I}) = 0 \]

\[ \hat{T} = \mu \hat{I} + \gamma \hat{N} \]

\[ \hat{T}^2 = 1 = \mu^2 + \gamma^2 + 2\mu\gamma \hat{I} \cdot \hat{N} \]

\[ \gamma = -\mu \cdot \hat{I} \cdot \hat{N} \pm \left(1 - \mu^2 \left(1 - (\hat{I} \cdot \hat{N})^2\right)\right)^{1/2} \]

Total internal reflection:

\[ 1 - \mu^2 (1 - (\hat{I} \cdot \hat{N})^2) < 0 \]
Optical Manhole

Total internal reflection

\[ n_w = \frac{4}{3} \]

From Livingston and Lynch

Fresnel Reflectance

Metal (Aluminum)

Gold \( F(0)=0.82 \)
Silver \( F(0)=0.95 \)

Dielectric (N=1.5)

Glass \( n=1.5 \) \( F(0)=0.04 \)
Diamond \( n=2.4 \) \( F(0)=0.15 \)

Schlick Approximation

\[ F(\theta) = F(0) + (1 - F(0))(1 - \cos \theta)^5 \]
Experiment

Reflections from a shiny floor

From Lafortune, Foo, Torrance, Greenberg, SIGGRAPH 97

Cook-Torrance Model for Metals

Reflectance of Copper as a function of wavelength and angle of incidence

Light spectra

Measured Reflectance

Approximated Reflectance

Copper spectra

Cook-Torrance approximation

\[ R = R(0) + R(\pi/2) \left( \frac{F(\theta) - F(0)}{F(\pi/2) - F(0)} \right) \]
**Ideal Diffuse Reflection**

Assume light is equally likely to be reflected in any output direction (independent of input direction).

\[
L_{r,d}(\omega_r) = \int f_{r,d}L_i(\omega_i)\cos \theta_i \, d\omega_i = f_{r,d}\int L_i(\omega_i)\cos \theta_i \, d\omega_i = f_{r,d}E
\]

\[
M = \int L_i(\omega_i)\cos \theta_i \, d\omega_i = L_i\int \cos \theta_i \, d\omega_i = \pi L_i
\]

\[
\rho_d = \frac{M}{E} = \frac{\pi L_i}{E} = \frac{\pi f_{r,d}E}{E} = \pi f_{r,d} \Rightarrow f_{r,d} = \frac{\rho_d}{\pi}
\]

**Lambert’s Cosine Law**

\[
M = \rho_d E = \rho_d E \cos \theta_i
\]

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**“Diffuse” Reflection**

**Theoretical**
- Bouguer - Special micro-facet distribution
- Seeliger - Subsurface reflection
- Multiple surface or subsurface reflections

**Experimental**
- Pressed magnesium oxide powder
- Almost never valid at high angles of incidence

**Paint manufacturers attempt to create ideal diffuse**
Phong Model

\[ (\hat{E} \cdot R(\hat{L}))^\circ \]

\[ (\hat{L} \cdot R(\hat{E}))^\circ \]

Reciprocity: \( (\hat{E} \cdot R(\hat{L}))^\circ = (\hat{L} \cdot R(\hat{E}))^\circ \)

Distributed light source!

Phong Model

Mirror

Diffuse
Properties of the Phong Model

Energy normalize Phong Model

\[
\rho(H^2 \rightarrow \omega_r) = \int_{H^2(S)} \left( \hat{L} \cdot R(\hat{E}) \right)^s \cos \theta \, d\omega_l \\
\leq \int_{H^2(R)} \left( \hat{L} \cdot R(\hat{E}) \right)^r \, d\omega_{ir} \\
\leq \int_{H^2} \cos^r \theta \, d\omega = \frac{2\pi}{s+1}
\]