CS 248: Introduction to Computer Graphics

Final Examination
Friday, March 21, 1997

This is exam is *closed-book.* No written materials or notes are to be used while taking this exam.

The exam consists of 6 questions. Each question is worth a different number of points. Please answer all the questions in the space provided, overflowing on to the back of the page if necessary. We estimate that this exam should take 1 hour, but you will have the entire 3 hours to complete the exam.

NAME:

SCORES:

1. Hidden surface (15)
2. BSP trees (20)
3. Viewing transformation (20)
4. Hierarchical models (20)
5. Splines (15)
6. Reflection (10)

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TOTAL SCORE :

Honor Code Statement

The Honor Code is an undertaking of the students, individually and collectively:

1. that they will not give or receive aid in examinations; that they will not give or receive unpermitted aid in class work, in the preparation of reports, or in any other work that is to be used by the instructor as the basis of grading;

2. that they do their share and take an active part in seeing to it that others as well as themselves uphold the spirit and letter of the Honor Code.

The faculty on its part manifests its confidence in the honor of students by refraining from proctoring examinations and from taking unusual and unreasonable precautions to prevent the forms of dishonesty mentioned above. The faculty will also avoid, as far as practicable, academic procedures that create temptations to violate the Honor Code. While the faculty alone has the right and obligations to set academic requirements, the students and faculty will work together to establish optimal conditions for honorable academic work.

I acknowledge and accept the Honor Code.

Printed name____

Signature____

NAME:

1. This question concerns modifications to scan conversion algorithms to perform hidden surface and shading calculations.

1A. [10 points] Outline how would you modify the triangle scan conversion procedure that you implemented in assignment 2 (the environment map viewer) so that it was capable of performing z-buffered hidden surface calculations. Explain what needs to be done at each vertex, at each edge, and at each pixel.

1B [5 points] Similarly, how would you modify the scan converter to shade once per pixel? Assume that shading involves recomputing the lighting and reflection from each point.

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2. An important data structure in computer graphics is a BSP Tree.

2A [5 points] What is a BSP Tree? Draw a diagram of a 2D BSP Tree and, referring to your diagram, describe the major features of the data structure?

2B [5 points] Given as input a collection of N polygons, describe an algorithm to create a BSP Tree?

2C [10 points] In class we described an algorithm that took as input the viewpoint and the BSP Tree, and output a list of polygons sorted in back to front order. A similar idea can be used to speed-up ray tracing. Suppose you are given a ray and a BSP Tree. How can you use the BSP Tree to find the intersection with the nearest polygon?

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3. [20 points] Viewing transformations. A common method for specifying the camera position is to give a look-from point **F**, a look-at point **A**, and an up-vector **U**.

Suppose

F = (1,0,1), **A** = (0,0,0) and **U** = (0,1,0)

Derive a 4x4 transformation matrix that may be used to view the world from this position. (Hint: make a drawing.)

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4. [20 points] Hierarchical models. You have just been hired by the Robotics Division to build a geometric model of the following simple robot arm.

The base of this robot arm is attached to the table. Consider the point of attachment to the origin of the world coordinate system. The 1st link of the arm is vertical, but may rotate about a vertical axis by an amount θ. At the end of this link is a joint that rotates about a perpendicular direction by ϕ. Finally, at the end of that link is a simple gripper, that consists of two "fingers" that rotate in opposite directions in concert by an angle α. Using transformations and predefined procedures to draw the various links and fingers, write down a sequence commands that can be used to draw the arm in its different positions as a function of θ, ϕ and α.

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5. [15 points] Interpolation is one of the most useful concepts in computer graphics. Interpolation is used in rendering to draw textures, in modeling to define smooth surfaces, and in animation to create inbetween frames from keyframes. This problem concerns a simple method for performing cubic interpolation.

Recall that in Hermite interpolation we are given the following 4 pieces of information:

$$h_0 = P(0)$$
$$h_1 = P(1)$$
$$h_2 = P'(0)$$
$$h_3 = P'(1)$$

Here h_0 and h_1 are the values at $t=0$ and $t=1$, and h_2 and h_3 are the derivatives or slopes. From this information, we derived a formula for the curve

$$P(t) = h_0 H_0(t) + h_1 H_1(t) + h_2 H_2(t) + h_3 H_3(t)$$

where $H_i(t)$ are the four cubic Hermite basis functions.

$$H_0(t) = 2t^3 - 3t^2 + 1$$
$$H_1(t) = -2t^3 + 3t^2$$
$$H_2(t) = t^3 - 2t^2 + t$$
$$H_3(t) = t^3 - t^2$$

Catmull and Rom had a clever idea. Suppose we are given two neighboring points on the curve

$$p_0 = P(-1)$$
$$p_1 = P(2)$$

We can then estimate the derivatives of the curve by at $t=0$ and $t=1$ by using central differences

$$h_2 = \frac{1}{2}(P(1) - P(-1))$$
$$h_3 = \frac{1}{2}(P(2) - P(0))$$

Thus, we now know values and derivatives at t=0 and t=1 and we can use Hermite interpolation to define a cubic interpolant in the interval from t=0 to 1. The result is the Catmull-Rom Spline

$$P(t) = p_0 C_0(t) + h_0 C_1(t) + h_1 C_2(t) + p_1 C_3(t)$$

Derive expressions for the polynomials $C_i(t)$. [Hint: Derive expressions for the C_i in terms of the H_i first.]

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6. Reflection [10 points] Suppose the surface of a material is roughened by tiny triangular grooves, all aligned in the same direction; and that the grooves act as perfect mirror reflectors.

Now suppose a planar swatch of this material is resting horizontally on the ground, and that the position of the sun is 45 degrees from the zenith (straight up). Describe the distribution of light reflected from the surface. What happens as you rotate the surface about its normal? In particular, what happens when the sun is aligned with the grooves vs. perpendicular to the grooves?

How would the appearance change if the wavy micro-surface was randomly perturbed (so that the normal was displaced slightly)?